

Evidence for cooling in an optical lattice by amplitude modulation

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We report on a generic cooling technique for atoms trapped in optical lattices. It consists in modulating the lattice depth with a proper frequency sweeping. This filtering technique removes the most energetic atoms and provides, with the onset of thermalization, a cooling mechanism reminiscent of evaporative cooling. However, the selection is here performed in quasimomentum space rather than in position space. Interband selection rules are used to protect the population with a zero quasimomentum, namely the Bose Einstein condensate. Direct condensation of thermal atoms in an optical lattice is also achieved with this technique. It offers an interesting complementary cooling mechanism for quantum simulations performed with quantum gases trapped in optical lattices.

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I. INTRODUCTION

Atomic and molecular physics have been strongly impacted by cooling techniques [1,2]. Laser cooling has triggered a boost of research activities to reach very low temperatures [3–5] and therefore to improve the control on the external degrees of freedom of atoms, with many applications in metrology [6,7]. Laser cooling down to degeneracy has been demonstrated only recently [8,9]. Those cooling techniques are tailor-made for a given species. Lasers shall indeed address specific atomic lines. Furthermore, the temperature achieved with laser cooling is strongly dependent on the width of the excited state of the considered cycling transition [10]. In contrast, evaporative cooling is a much more generic technique. The filtering technique of the most energetic atoms can be easily transposed from one species to another and the cooling occurs through the rethermalization of the atomic cloud. Only the cooling rate depends on the species [11–18].

Currently, quantum gases in optical lattices are used to perform quantum simulations and temperature may constitute an obstacle for some challenging experiments [19,20]. It is therefore of utmost importance to find methods to decrease the temperature in such an environment. The Raman sideband cooling provides an elegant possibility [9,21–23]. However, this latter method requires the manipulation of the internal state of the atoms. In this article, we report on a generic cooling method which does not involve the use of the internal state and that is inspired by evaporative cooling but adapted to atoms trapped in an optical lattice [24].

In this article, we exploit interband transitions excited by amplitude modulation for nonzero quasimomentum [25]. The modulation frequency is scanned through all the values of the considered interband transition. The chosen excited band obeys a selection rule from the ground-state band which prohibits the transition of the wave-function component with zero quasimomentum. As a result atoms with the lowest quasimomentum are not concerned by the excitation. In the following, we first present the experimental setup. We then show experimentally the effectiveness of selection rules. We

also explain how parameters shall be tuned in order to benefit from the filtering operated by the frequency scan to remove the thermal wings of a partially condensed atomic cloud. The collisional dynamics which takes place inside the cloud after the removal of the most energetic atoms yields an increase of the fraction of condensed atoms and a concomitant gain in phase-space density. The method is eventually demonstrated on thermal atoms that are cooled down to degeneracy in this manner.

II. EXPERIMENTAL SETUP AND PROTOCOL

Our experiment starts with a ^{87}Rb Bose Einstein condensate (BEC) realized in a hybrid trap made of a quadrupole magnetic trap and two intersecting far-from-resonance optical dipole beams (wavelength 1064 nm, one horizontal and the other vertical) [26,27]. This setup can produce nearly pure condensates of about 10^5 atoms. However, in this article, we have used uncondensed or partially condensed atomic cloud. We control the degree of degeneracy through the duration of the evaporative cooling ramp. Before loading the atoms into the optical lattice, we remove adiabatically the vertical beam of the dipole trap; we then ramp up in 30 ms a one-dimensional optical lattice realized by the interference of two counterpropagating laser beams along the x axis (yielding a lattice spacing of $d = 532$ nm) aligned with the horizontal dipole trap. The optical lattice has a depth, V_0 , measured through the dimensionless parameter s_0 : $V_0 = s_0 E_R$, where $E_R = \hbar^2 / (8md^2)$ is the recoil energy. The horizontal extra confinement provided by the dipole beam and the magnetic gradient of the hybrid trap is equivalent to a harmonic confinement of frequency 5 Hz.

Once the sample has been prepared, we apply an amplitude modulation of the lattice whose frequency is scanned in resonance with some excited bands of the static optical lattice. During the modulation which lasts a time duration t_{mod} , the lattice potential experienced by the atoms reads

$$V(x, t) = -s_0 E_R [1 + \varepsilon(t)] \cos^2\left(\frac{\pi x}{d}\right). \quad (1)$$

The modulation of the lattice depth is $\varepsilon(t) = \varepsilon_0 \sin[2\pi\nu(t)t]$ with the frequency chirp $\nu(t) = \nu_i + (\nu_f - \nu_i)t/2t_{\text{mod}}$. The amplitude ε_0 is slightly larger than 0.3 (see below). The instantaneous frequency $\nu(t)$ varies slowly in time with respect to the period of oscillation $1/\nu(t)$: a typical modulation time is on the order of 10 ms, while the typical modulation frequency is on the order of 10 to 30 kHz.

The principle of the method is easily explained by assuming that the initial atomic cloud lies in the ground-state band after the adiabatic loading into the optical lattice. It spreads on a continuum of Bloch states whose quasimomenta are in the interval $k \in [-\Delta k/2; \Delta k/2]$. The cooling method that we propose in the following starts by promoting atoms with more energy than the mean energy to higher bands using resonant amplitude modulation [25] [the coupling strength between bands n and p is proportional to the matrix element $V_{np} = |\langle n, k | \cos^2(\pi x/d) | p, k \rangle|^2$, where $|n, k\rangle$ are the Bloch states]. Those atoms are associated to the largest quasimomenta of the interval of k . Once promoted they propagate along the lattice with a velocity dictated by the local slope dE/dk of the excited band which is on the order of a few hundreds of micrometers per second [28]. Those excited atoms ultimately leave the sample. In this procedure, we filter out the most energetic atoms. To obtain a gain in phase space and/or recover equilibrium we rely on elastic collisions.

The robustness of our method originates from the use of selection rules and therefore from the proper choice of bands over which the excitation is performed. Indeed, the selection rules for amplitude modulation at the center of the Brillouin zone ($k = 0$) inhibit the promotion of atoms from the ground-state band to the first [see Fig. 1(a)] and third excited bands, and from the first excited band to the fourth one [25,29]. However, this rule is only valid at the center of the Brillouin zone. We exploit this interesting feature to avoid the excitation of atoms from the condensate when we apply our protocol on a partially condensed cloud of atoms. Indeed, in this latter case, the condensate occupies a narrow range of k about $k = 0$ in the ground-state band, while the thermal part spans a larger k interval on the two lowest bands (see below). Our method relies on a precise knowledge of the band structure and the lattice depth, accurately determined using the shift method [27].

III. BEC PROTECTION BY SELECTION RULES

Before addressing cooling issues, we directly demonstrate the efficiency of the protection of the BEC against the modulation. For this purpose, the experiment is carried out on a cloud with a condensed fraction of 40% in a modulated lattice of depth $s_0 = 10.8$ and with a modulation amplitude, $\varepsilon_0 = 0.38$. The excitation from the ground-state band to the first excited band occurs in the frequency range of 9.9(1) to 11.9(1) kHz and the lowest frequency to excite the second band is $\nu_{13} = 16.7(2)$ kHz for our parameters. We scan the modulation frequency from $\nu_i = 11$ kHz to an adjustable final frequency ν_f [see Fig. 1(c)] chosen in the frequency range of 11.2 to 17.6 kHz. To get rid of the atoms that have been promoted to the excited bands, we accelerate them by ramping down the optical lattice in 25 ms, i.e., we use the band mapping technique [30–32]. Atoms then evolve in the horizontal guide for $t_{\text{hold}} = 20$ ms, and an absorption image is taken after

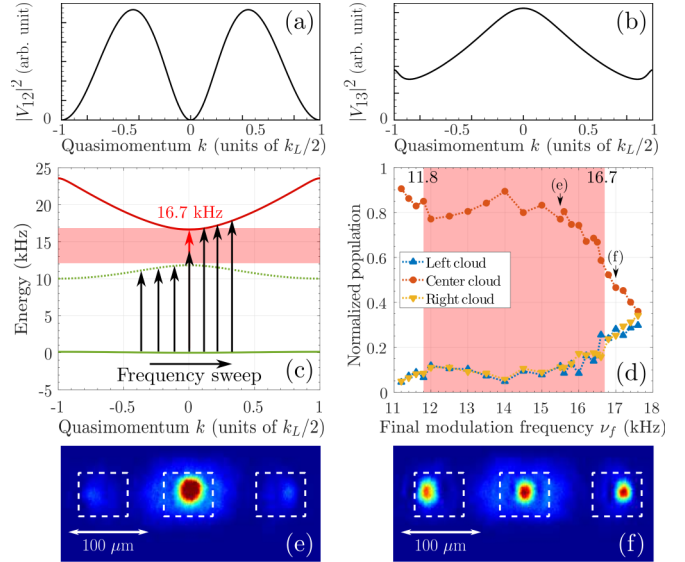


FIG. 1. (a),(b) Strength of the couplings, $|V_{12}|^2$ and $|V_{13}|^2$, between the ground state and the excited bands as a function of the quasimomentum k in units of $k_L = 2\pi/d$. (c) Range of frequency spanned during the amplitude modulation superimposed to the band structure (the pink area corresponds to the gap). Bound (unbound) bands are depicted in green (red). Dashed green band is not excited at $k = 0$ by amplitude modulation. (d) Number of atoms in the white dashed boxes of the absorption images [see for example (e) and (f)] for a frequency sweep of the amplitude modulation from $\nu_i = 11$ kHz to an adjustable final frequency, ν_f , ranging from 11.2 to 17.6 kHz. Absorption images for $\nu_f = 15.5$ kHz (e) and $\nu_f = 17$ kHz (f). Parameters: $s_0 = 10.8$, $\varepsilon_0 = 0.38$, and $t_{\text{mod}} = 6$ ms.

a 25 ms time of flight in the absence of any confinement. Two examples of such absorption images are depicted in Figs. 1(e) and 1(f). Figure 1(e) was taken after the frequency sweep with final frequency $\nu_f = 15.5$ kHz, a frequency below the minimum frequency, ν_{13} , required to excite the second band. We observe two lateral clouds that correspond to the excited atoms leaving the central cloud with a finite velocity. Interestingly, the frequency $\nu(t)$ has been resonant with the $k = 0$ transition [11.9(1) kHz] from the ground state to the first excited band but we do not see any atoms from the condensate in the lateral clouds. This is to be contrasted with what happens when the final frequency ν_f is above ν_{13} . In this case, atoms from the condensate at $k = 0$ are promoted to the second excited band for which there is no selection rules that forbid the excitation of the $k = 0$ component [see Figs. 1(a) and 1(b)]. This can be clearly seen on the absorption image of Fig. 1(f) taken for $\nu_f = 17$ kHz: the lateral clouds contain Bose-condensed atoms. Furthermore, we observe that the atom number in the central cloud decreases while the final frequency increases in the interval resonant with the first band [see Fig. 1(d)]. It stays nearly constant in the gap except at the close proximity below the frequency ν_{13} . This latter feature may be associated to the fluctuations of the lattice depth from shot to shot and/or to the modification of the band structure that results from the resonant coupling between the ground and first excited bands.

IV. EVIDENCE FOR COOLING

This first experiment confirms the effectiveness of the selection rules when amplitude modulation is applied. However, atoms that are promoted to the first excited band move towards the edges of the cloud quite slowly (on the order of $10 \mu\text{m/s}$ for our parameters) at a velocity governed by the slope of the dispersion relation. Furthermore, those atoms cannot strictly speaking escape from the lattice envelope since they are propagating on a bound band [33,34] [see Fig. 1(c)]. We have therefore performed a second set of experiments where we promote atoms to higher excited bands that are unbound. The choice of the excited band results from a trade-off. We want the promoted atoms to belong to an unbound band but for a given amplitude of modulation ε_0 , the probability of excitation decreases with the band index. We have finally chosen to carry out a quantitative study of the cooling mechanism by using resonant transitions from the ground state to the third excited bands with a bare lattice of depth $s_0 = 8.8$ and a modulation amplitude $\varepsilon_0 = 0.38$.

We have chosen partially condensed atomic clouds with an initial temperature of $130(10)$ nK. This energy scale is to be compared with the energy width of the ground-state band at a vanishing lattice depth equal to $\hbar^2/8md^2$, i.e., 97 nK. Through the adiabatic loading, the uncondensed cloud will thus populate completely the ground-state band and partially the first excited band. The modulation frequency varies from $\nu_i = 25$ kHz to a final modulation frequency ν_f ranging from 27 to 37.5 kHz. Interestingly, this range of frequencies is resonant for all quasimomenta of the ground band to the third excited band, *and also* can promote atoms of the first excited band to the fourth one but only in the quasimomenta interval $-0.3 k_L$ to $0.3 k_L$, where $k_L = 2\pi/d$.

Figure 2 summarizes our results on the atoms that are not excited by the modulation, analyzed after ramping down the lattice. We have represented as a function of ν_f the corresponding number of thermal atoms, the number of condensed atoms, and the temperature. Data are taken alternatively with and without modulation to check the reproducibility. For the data recorded in the absence of modulation, the mean value accompanied with its variance is depicted as a horizontal line and blue zone in Figs. 2(b). We clearly observe that the number of condensed atoms is roughly not affected by the amplitude modulation up to a final modulation frequency close to the resonant frequency $\nu_{15} = 36.3$ kHz between the ground-state band and the fourth excited band at the center of the Brillouin zone, where no selection rules protect the BEC. By contrast the number of thermal atoms continuously decreases. Our method clearly demonstrates that the most energetic atoms in the lattice can be extracted from the lattice. This is an important result for realizing quantum simulations with optical lattices.

The ‘‘instantaneous’’ temperature measured using the standard bimodal fit is clearly reduced for small final modulation frequency, a phenomenon that can be associated to the removal of the most energetic atoms as expected. However, in an interval close to the resonant frequency the temperature starts to increase even above the initial temperature, the number of condensed atoms decreases ($\nu_f > 35$ kHz), since the modulation amplitude removes atoms with less energy than

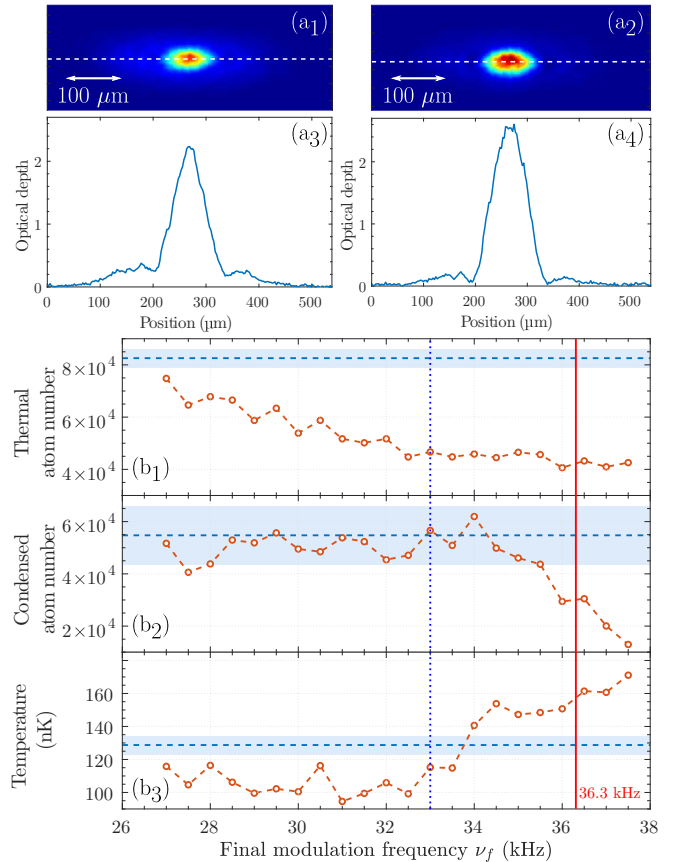


FIG. 2. Experimental results. The modulation frequency is scanned from $\nu_i = 25$ kHz to an adjustable ν_f ranging from 27 to 37.5 kHz and resonant with the third excited band. (a) Example of results: (a₁) Absorption image of a cloud of 3.4×10^4 condensed atoms and 8.8×10^4 thermal atoms at a temperature $T = 115$ nK that has experimented the protocol except for the modulation. (a₂) Absorption image obtained with the modulation protocol for $\nu_f = 33$ kHz [vertical blue dashed line in (b)]. (a₃),(a₄) Cut of both images along the white dashed line. (b) Summary of all similar data: the number of thermal atoms (b₁) and condensed atoms (b₂) as well as the temperature (b₃) are depicted as a function of ν_f . Vertical red line identifies the frequency $\nu_{15} = 36.3$ kHz of the transition to the fourth excited band at $k = 0$ for the bare lattice. For each subgraph (b), the horizontal blue dashed line corresponds to the mean value obtained with the same protocol except that no modulation is applied; the blue zone about the mean value reflects the variance of the mean value. Parameters: $s_0 = 8.8$, $\varepsilon_0 = 0.38$, and $t_{\text{mod}} = 10$ ms.

the mean energy. This means that our filtering protocol is accompanied by a thermalization process. The interpretation of the temperature should nevertheless be taken with care since the system is not yet thermalized (thermal atoms have undergone less than two collisions per atom in average).

V. DIRECT BOSE EINSTEIN CONDENSATION

As a result it shall be possible to use directly this protocol to increase the phase-space density of the original cloud. For this purpose, we did another set of experiments where we stopped the evaporative cooling just above the condensation threshold in the hybrid trap. The vertical beam of the dipole

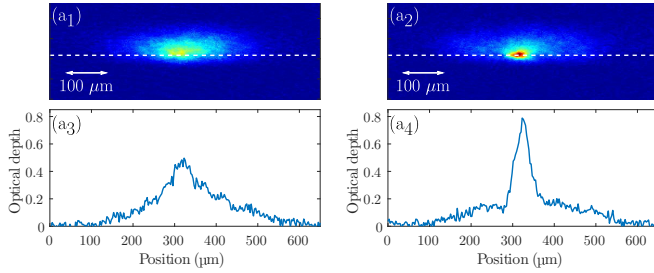


FIG. 3. (a₁) Absorption image of a cloud of 7×10^4 uncondensed atoms at a temperature $T = 160$ nK that has experimented the different stages of the protocol except for the modulation. (a₂) Absorption image obtained when applying the amplitude modulation protocol (see text). The cloud is now partially condensed (2.8×10^3 condensed atoms and 5.6×10^4 thermal atoms). (a₃),(a₄) Cut of both images along the white dashed line.

trap is subsequently ramped down. At this stage, we have a cloud of 7×10^4 atoms at a temperature $T = 160$ nK. For this experiment, we have chosen the following parameters: $s_0 = 7.2$, $\varepsilon_0 = 0.32$, and $t_{\text{mod}} = 10$ ms. The frequency was tuned from $\nu_i = 25.7$ kHz to $\nu_f = 33.9$ kHz, driving the transition from the ground-state band to the third excited band and from the first to the fourth excited bands. To analyze the data, we proceed as previously where atoms first propagate after the adiabatic ramping down of the lattice during $t_{\text{hold}} = 25$ ms into the horizontal guide of the dipole trap before the switching off of all confinements. The absorption images presented in Fig. 3 are taken after a 25 ms time of flight. For the same confinement, we observe the emergence of a small condensate of 2.8×10^3 atoms surrounded by a thermal cloud of 5.6×10^4 atoms. The reproducibility of this experiment was confirmed by alternating measurements performed with and without modulation.

VI. DISCUSSIONS AND CONCLUSION

This experiment confirms that our amplitude modulation protocol acts as evaporative cooling: a fraction of the most energetic atoms are filtered out by the modulation and the thermalization takes place. There is however a few important differences. Indeed the most energetic atoms selected by the modulation are promoted to excited bands whatever is their original position inside the lattice since the selection is made on the quasimomentum. As a result, those atoms have to cross the sample and may collide with other atoms. Inside the lattice, the excited atoms have a velocity governed by the slope of the excited band over which they lie [28]. If this velocity is below the superfluid velocity one expects the excited atoms to cross the BEC without depositing any energy [35]. The sound

velocity is estimated to 3 mm/s and the maximum velocity of the excited state is on the order of $100 \mu\text{m/s}$ in our case. However, in the band mapping procedure (adiabatic ramping down of the lattice), excited atoms are accelerated [25] above the sound velocity and dissipation may occur. Heating effects due to intra- and interband transitions assisted by two particle processes are known to be problematic for a 1D modulated lattice since collisions can deposit energy in the transverse degrees of freedom [36–43]. In our 1D modulated lattice, we have observed a clear cooling effect up to a lattice depth $s_0 = 16$, no more observable cooling effect at $s_0 = 28$, and even heating for a too long modulation time (80 ms).

In conclusion, we have demonstrated an experimental technique that enables one to remove the most energetic atoms from an optical lattice. This method amounts to engineering the populations among the bands. The method shown in this article is generic and could be applied to cold fermions. Indeed, in this latter case, the possibility to empty the excited bands using a frequency scan of the amplitude modulation enables one to generate an insulator in 1D with a completely filled lowest band. This method can also be used to study out-of-equilibrium dynamics by removing selectively some classes of excited atoms. We have shown that this technique can be used to increase the phase-space density of the trapped cloud thanks to the collisional dynamics. In this perspective, it is worth noticing that the cooling technique presented here is, to some extent, reminiscent of the velocity-selective coherent population trapping (VSCPT) cooling scheme using a pair of Raman beams [44]. In the latter, the zero momentum velocity class is immune to the Raman light, while other velocity classes undergo a Rabi oscillation between the coupled states that are contaminated by the excited state. As a result, atoms can decay by spontaneous emission. The corresponding randomization of the momentum can lead some atoms into the zero momentum class where they accumulate. In our protocol, most energetic atoms are removed. The subsequent thermalization increases the population by bosonic amplification in the phase-space cell of the BEC, a state that is immune to amplitude modulation thanks to selection rules. Interestingly, one could cycle the protocol to get an even stronger cooling effect. It therefore offers a complementary cooling mechanism dedicated to quantum simulations performed in optical lattices.

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