Shortcut to Adiabatic Passage in Two- and Three-Level Atoms

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We propose a method to speed up adiabatic passage techniques in two-level and three-level atoms extending to the short-time domain their robustness with respect to parameter variations. It supplements or substitutes the standard laser beam setups with auxiliary pulses that steer the system along the adiabatic path. Compared to other strategies, such as composite pulses or the original adiabatic techniques, it provides a fast and robust approach to population control.

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Introduction.—Two major routes for manipulating the state of a quantum system with interacting fields are the use of (i) resonant pulses and (ii) adiabatic methods such as ''rapid'' adiabatic passage (RAP), stimulated Raman adiabatic passage (STIRAP), and their variants. In general terms, simple fixed-area resonant pulses may be fast if intense enough, but unstable with respect to errors or fluctuations of the parameters, whereas adiabatic passage is robust but slow. For many applications, from NMR to quantum information processing, the ideal method should be fast and robust, combining the best of the two worlds. These two requirements are particularly demanding if quantum computing is to become feasible at all. Pulse sequences may be more stable than single pulses, but their use is limited by the longer times required, the need to control phase angles and pulse durations accurately, or off-resonant excitations due to sharp pulse edges [1]. Moreover, the error compensating properties of square-pulse sequences are not preserved when substituting them with smooth pulses so that the design of good sequences requires ''a good portion of experience and magic'' [2]. In NMR, composite pulses are being superseded by adiabatic passage methods [1], which have also been very successful in chemical reaction dynamics [3], laser cooling, atom optics [4], metrology [5], interferometry [6], or cavity quantum electrodynamics [7]. When robustness is the primary concern, they are quite sufficient, and have as well become basic operations for quantum information processing, to design robust gates [8,9], or in quantum adiabatic computing [10,11]. If speed is also important, however, the limitations may be severe [11]. Given the difficulties of composite pulses, it is natural to look for robustness and high operation rates taking instead the adiabatic methods as the starting point and shortening their duration somehow. We propose here a shortcut to adiabatic passage (''SHAPE'' hereafter) using the ''transitionless quantum driving'' algorithm by Berry [12]. The specific applications we shall discuss are speeded-up versions of (two-level) RAP and (three-level) STIRAP, as canonical examples of other adiabatic methods. Variants such as fractional RAP or STIRAP, and multilevel schemes, may be treated similarly.

The transitionless quantum driving algorithm [12] provides Hamiltonians $\mathcal{H}(t)$ for which the adiabatic approximation for the time-dependent wave function evolving with a reference Hamiltonian $H_0(t)$ becomes exact. The simplest Hamiltonian, $H_1(t)$, steers the dynamics along the instantaneous eigenstates $|\lambda_n(t)\rangle$ of $H_0(t)$ without transitions among them and without phase factors, formally in an arbitrarily short time,

$$H_1(t) = i\hbar \sum_n |\partial_t \lambda_n\rangle\langle\lambda_n|. \tag{1}$$

As far as populations are concerned, the addition of H_0 is possible, but not necessary, affecting only the phases. Thus H_1 may supplement H_0, when $\mathcal{H} = H_0 + H_1$, or substitute it, when $\mathcal{H} = H_1$. At variance with Lyapunov-control methods [13], $\mathcal{H}$ is independent of the time-dependent state so it leads to simpler, linear dynamics, and provides systematically exact solutions for adiabatic following without the need for a trial and error approach [13]. The price to pay is that information on the instantaneous eigenstates is required, but this is not problematic in many cases. The physical realizability of H_1 needs a separate study in each system. For example, when H_0 describes a particle in a time-dependent harmonic potential, H_1 becomes a nonlocal interaction [14]. (For local-interaction solutions, see [15–17].) For a particle with spin in a time-dependent magnetic field, H_1 becomes a time-dependent magnetic field [12]. For the atomic two- and three-level systems studied here, H_1 will involve auxiliary laser or microwave interactions. The optional addition of H_0 will imply different physical implementations.

Rapid adiabatic passage.—Let us consider first the speeding up of a standard rapid adiabatic passage that inverts the population of two levels, $|1\rangle$ and $|2\rangle$, by sweeping the radiation through resonance. This broad spread technique originated in NMR [18] but is used in virtually all fields where two-level systems may be controlled by external interactions, such as laser chemistry, modern quantum optics, or quantum information. When the frequency sweep is shorter than the lifetime for spontaneous emission and other relaxation times, it is termed rapid adiabatic passage.

H_0, H_1, or $\mathcal{H}$ will be formulated in a (quasi-)interaction picture here so they will carry an I superscript, to distinguish them from Schrödinger picture Hamiltonians (with S subscript). This will also apply to wave functions. To set H_0^I let us assume a semiclassical description of the (electric dipole) interaction with the electric field $\mathbf{E}$. For alkali atoms the Rabi frequency is given by $\Omega_R = \langle 2|e\mathbf{r}|1\rangle \cdot \mathbf{E}/\hbar$, $\mathbf{r}$ being the vector position operator of the valence electron. Using the rotating wave approximation (RWA), the Hamiltonian for a laser interaction with linear polarization in x direction is $H^S = \frac{\hbar}{2}[|2\rangle\langle 1|\Omega_R e^{-i\omega_L t} + |1\rangle\langle 2|\Omega_R e^{i\omega_L t} + \omega_0(|2\rangle\langle 2| - |1\rangle\langle 1|)]$, with Ω_R real here and hereafter. In a laser-adapted quasi-interaction picture based on $h_0(t) = \frac{\hbar\omega_L(t)}{2}(|2\rangle\langle 2| - |1\rangle\langle 1|)$, the dynamics of the wave function $\psi^I(t) = e^{ih_0 t/\hbar}\psi^S(t)$ is governed by the Hamiltonian $H_0^I = e^{ih_0 t/\hbar}(H^S - h_0 - t\dot{h}_0)e^{-ih_0 t/\hbar}$, where the dot denotes time derivative. (In a regular interaction picture the evolution operator for the time-dependent h_0 should be applied. Using instead the exponential $e^{-ih_0 t/\hbar}$ provides a simpler and exact quasi-interaction picture, thanks to the commutation between h_0 and $\dot{h}_0$.) Using $|1\rangle = \binom{1}{0}$, $|2\rangle = \binom{0}{1}$,

$$H_0^I(t) = \frac{\hbar}{2}\begin{pmatrix} -\Delta(t) & \Omega_R(t) \\ \Omega_R(t) & \Delta(t) \end{pmatrix}, \tag{2}$$

where $\Delta(t) = \omega_0 - \omega_L - t\dot{\omega}_L$ is the effective detuning, controlled by a change in the carrier frequency or an alteration of the Bohr frequency by Zeeman or Stark shifts. Note the inverse relation, $\omega_L = \omega_0 + \frac{1}{t}\int_0^t \Delta(t')dt'$. The instantaneous eigenvectors are $|\lambda_-(t)\rangle = \cos[\theta(t)/2]|2\rangle - \sin[\theta(t)/2]|1\rangle$ and $|\lambda_+(t)\rangle = \sin[\theta(t)/2]|2\rangle + \cos[\theta(t)/2]|1\rangle$, with the mixing angle $\theta(t) \equiv \arccos[-\Delta(t)/\Omega(t)]$ and eigenvalues $E_\mp(t) = \mp\hbar\Omega/2$, where $\Omega = \sqrt{\Delta^2(t) + \Omega_R^2(t)}$. If the adiabaticity condition $\frac{1}{2}|\Omega_a| \ll |\Omega(t)|$, where $\Omega_a \equiv \dot{\theta} = [\Omega_R(t)\dot{\Delta}(t) - \dot{\Omega}_R(t)\Delta(t)]/\Omega^2$, is satisfied, the state evolving from $|\psi^I(t_i)\rangle = |\lambda_\pm(t_i)\rangle$ follows the adiabatic approximation $|\psi_\pm^I(t)\rangle = \exp\{-\frac{i}{\hbar}\int_{t_i}^t dt' E_\pm(t')\}|\lambda_\pm(t)\rangle$, whereas transitions will occur otherwise. Different adiabatic passage schemes correspond to $\Omega_R(t)$ and $\Delta(t)$ for which $\psi_\pm^I$ goes from one bare state to the other. The simplest one is the Landau-Zener scheme with constant Ω_R and linear-in-time Δ. For the examples below we shall use the more adiabatic (and potentially faster) Allen-Eberly (AE) scheme [19,20]: $\Omega_R(t) = \Omega_0\,\mathrm{sech}(\pi t/2t_0)$, $\Delta(t) = (2\beta^2 t_0/\pi)\tanh(\pi t/2t_0)$. Regardless of the scheme chosen, $H_1^I(t)$ here takes the form, using Eq. (1),

$$H_1^I(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & -i\Omega_a \\ i\Omega_a & 0 \end{pmatrix}, \tag{3}$$

where (up to a phase factor) Ω_a plays the role of the Rabi frequency for a fast-driving field. In principle, H_1^I drives the dynamics along the H_0^I-adiabatic path in arbitrarily short times, but there are practical limitations, such as the laser power available. Moreover, a comparison with H_0^I dynamics is only fair if $|\Omega_a|$ is smaller or approximately equal to the peak Rabi frequency in the original laser setup. Independently of the scheme chosen and in a range of interaction times that break down the adiabaticity condition, it is remarkable that the dynamics can be driven along the H_0^I-adiabatic path while fulfilling the inequalities $|\Omega_a| \leq |\Omega| \leq |\Omega_0|$.

The physical meaning and realizability of the fast-driving term are determined by going back to the Schrödinger picture: For $\mathcal{H}^I = H_0^I + H_1^I$, then $\mathcal{H}^S = \frac{\hbar}{2}[(\Omega_R + i\Omega_a)|2\rangle\langle 1|e^{-i\omega_L t} + (\Omega_R - i\Omega_a)|1\rangle\langle 2|e^{i\omega_L t} + \omega_0(|2\rangle\langle 2| - |1\rangle\langle 1|)]$. This implies two lasers with the same frequency, orthogonal polarization, and differently shaped time-dependent intensities. Instead, when $\mathcal{H}^I = H_1^I$, $\mathcal{H}^S = \frac{\hbar}{2}[i\Omega_a|2\rangle\langle 1|e^{-i\omega_L t} - i\Omega_a|1\rangle\langle 2|e^{i\omega_L t} + \omega_L(|2\rangle\langle 2| - |1\rangle\langle 1|)]$, which requires only one interaction and level shift engineering so that $\Delta = 0$. In this case the $\pm i$ factors in Eq. (3) can be dropped, which amounts to redefining the states with constant phase factors or to performing an axis rotation, without altering the population transfer.

For the AE scheme the population of the excited state P_2 starting from the ground state depends on the dimensionless parameters $\tau = t_0\beta$ and $\omega = \Omega_0/\beta$ [20]: $P_2 = 1 - \mathrm{sech}^2(2\tau^2/\pi)\cos^2[\tau(\omega^2 - 4\tau^2/\pi^2)^{1/2}]$. A population transfer near to 1 ($P_2 > 0.999$) and stable with respect to parameter variations is achieved for $\omega \geq 3$ and $\tau \geq 3$. We may calculate Ω_a and the minimal time for which the maximum of $|\Omega_a|$ with respect to t is $\leq \Omega_0$. In the stated range this is accurately given by $\tau_m = \pi/(4\omega)$, or $t_{0,m} = \pi/(4\Omega_0)$. The reduction factor with respect to the adiabatic time $\tau_a \approx 3$ may be very significant, $t_{0,m}/t_a \approx \pi/(12\omega)$; this is 0.09 for $\omega = 3$ or 0.01 for $\omega = 20$. Of course the SHAPE Hamiltonians $\mathcal{H}^I$ may also drive the system along the adiabatic path outside the $w, \tau > 3$ domain as illustrated in Fig. 1.

For comparison, the population of the excited state due to a square π pulse with on-resonance Rabi frequency Ω_0 is $P_2 = \frac{\Omega_0}{\Omega}|\sin(\frac{\Omega t}{2})|^2$. Complete population transfer requires $\Delta = 0$ and a pulse time $t_R = \frac{\pi}{\Omega_0}$. For the same Ω_0 and limiting the auxiliary laser by $|\Omega_a| \leq \Omega_0$, the minimal characteristic time $t_{0,m}$ of the SHAPE method is

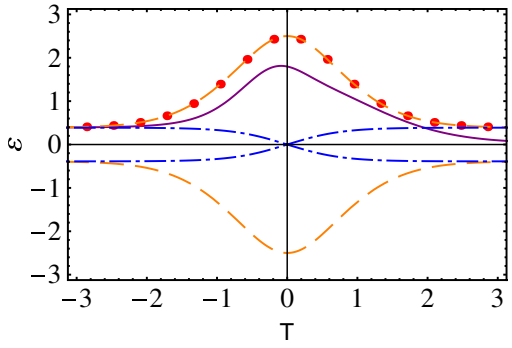

FIG. 1 (color online). Dimensionless energies (in quasi-interaction picture) $\varepsilon = E/(\beta\hbar)$ versus dimensionless time $T = t\beta$ for the AE scheme: diabatic energies (dot-dashed blue lines), adiabatic energies (dashed orange lines), average energy evolving with H_0^I (solid purple line), average energy evolving with $\mathcal{H}^I = H_1^I$ or $\mathcal{H}^I = H_0^I + H_1^I$ (dotted red line), indistinguishable from the adiabatic energy. Parameters: $\omega = 5$ and $\tau = 1.22$.

$t_{0,m} = t_R/4$. The actual interaction time to implement a successful population inversion with the AE scheme (SHAPE corrected or not) should be a few times t_0; this may be estimated from the dependence of the excited population of the adiabatic state with time, which is >0.999 for $t \geq 8t_0$. All calculations are performed by solving numerically the dynamical equation of ψ^I by a Runge-Kutta method with adaptive step size control.

Figure 2 shows examples of the fidelity ($F = P_2$) with respect to variations in the Rabi frequency and detuning with SHAPE (AE scheme for H_0^I), the evolution with H_0^I (AE scheme), a Rabi π pulse, and a composite $\frac{\pi}{2}(x)\pi(y) \times \frac{\pi}{2}(x)$ pulse, a fault-tolerant combination where x, y refer to the laser polarization (and Pauli matrix) involved. Clearly SHAPE provides a fast, robust, and efficient population inversion compared to all other methods. All cases are for the same Ω_0, and in SHAPE $|\Omega_a| \leq \Omega_0$.

Stimulated Raman adiabatic passage.—Similar ideas can be applied to three-level STIRAP. The (RWA) Hamiltonian for the two-photon resonance case in a laser-adapted (ordinary) interaction picture reads [7]

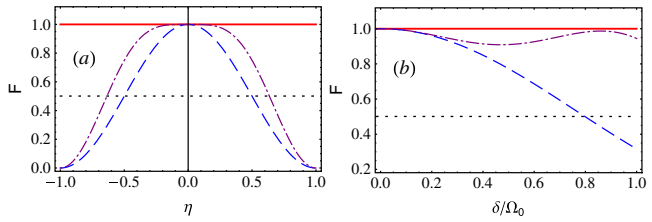

FIG. 2 (color online). Fidelity ($F = P_2$) versus the changes of (a) the Rabi frequency from Ω_0 to $\Omega_0(1 + \eta)$, (b) the detuning by δ. SHAPE, AE scheme (solid red line) with $\Omega_a \leq \Omega_0$, ordinary AE adiabatic passage (dotted black line), Rabi pulse (dashed blue line), composite pulse $\frac{\pi}{2}(x)\pi(y)\frac{\pi}{2}(x)$ (dot-dashed purple line). $\Omega_0 = 2\pi \times 5$ MHz, $t_0 = 25$ ns, $\beta = 2\pi$ MHz.

$$H_0^I(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 2\Delta & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{pmatrix}, \tag{4}$$

in terms of the Rabi frequencies for the Stokes, $\Omega_s(t)$, and pumping lasers, $\Omega_p(t)$, and the (now constant) laser detuning $\hbar\Delta = (E_2 - E_1) - \hbar\omega_p = (E_2 - E_3) - \hbar\omega_s$, ω_p and ω_s being the corresponding laser frequencies. The instantaneous eigenstates $|\lambda_n\rangle$ are $|\lambda_+(t)\rangle = \sin\theta\sin\phi|1\rangle + \cos\phi|2\rangle + \cos\theta\sin\phi|3\rangle$, $|\lambda_-(t)\rangle = \sin\theta\cos\phi|1\rangle - \sin\phi|2\rangle + \cos\theta\cos\phi|3\rangle$, and $|\lambda_0(t)\rangle = \cos\theta|1\rangle - \sin\theta|3\rangle$, with eigenvalues $E_+(t) = \hbar\Omega\cot(\phi/2)$, $E_0 = 0$, and $E_-(t) = -\hbar\Omega\tan(\phi/2)$. The mixing angles θ and ϕ are, respectively, defined by $\tan\theta = \Omega_p(t)/\Omega_s(t)$ and $\tan(2\phi) = \Omega/\Delta(t)$, whereas now $\Omega = \sqrt{\Omega_p^2(t) + \Omega_s^2(t)}$. The population transfer $1 \rightarrow 3$ is realized by the "dark state" $|\lambda_0\rangle$. The Hamiltonian $H_1^I(t)$, see Eq. (1), takes the form

$$H_1^I(t) = i\hbar\begin{pmatrix} 0 & \dot\phi\sin\theta & \dot\theta \\ -\dot\phi\sin\theta & 0 & -\dot\phi\cos\theta \\ -\dot\theta & \dot\phi\cos\theta & 0 \end{pmatrix}, \tag{5}$$

with $\dot\theta = [\dot\Omega_p(t)\Omega_s(t) - \dot\Omega_s(t)\Omega_p(t)]/\Omega^2$ and $\dot\phi = \{[\dot\Omega_p(t)\Omega_p(t) + \dot\Omega_s(t)\Omega_s(t)]\Delta(t)\}/[2\Omega(\Delta^2 + \Omega^2)]$. We would need, in principle, three new interactions to implement this Hamiltonian. The laser fields connecting levels 1-2 and 2-3 should have the same frequency as the original ones, and the field connecting levels 1-3 should be on resonance with this transition. (The RWA approximation is assumed in all cases.) If the transition 1-3 is electric-dipole forbidden, as it is the case for hyperfine levels of alkali atoms, a magnetic dipole transition may be used instead, although this may limit the intensity and thus the ability to shorten the times. By working in the adiabatic basis we see that $d\langle\lambda_0(t)|\psi^I(t)\rangle/dt$ does not depend on $\dot\phi$ for an arbitrary ψ^I, so that the 1-3 and 2-3 auxiliary fields do not affect $\langle\lambda_0(t)|\psi^I(t)\rangle$ and are thus not necessary for a full passage from 1 to 3. H_1^I may therefore be simplified to

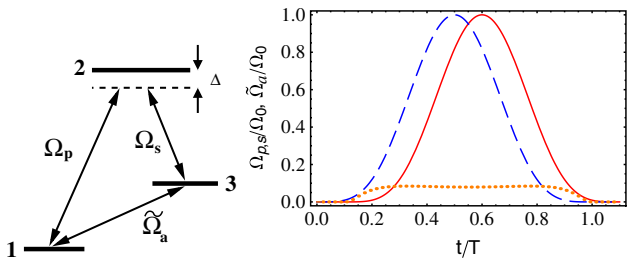

FIG. 3 (color online). Level scheme (left) and evolution of Rabi frequencies (right) in a STIRAP sequence of laser pulses, Eq. (7), with $\Omega_0 = 2\pi \times 2$ MHz, $\Delta = 2\pi \times 0.1$ MHz, and $T = 4$ μs, $\tau = 0.1T$. Ω_s/Ω_0 (dashed blue line), Ω_p/Ω_0 (solid red line), $\tilde\Omega_a/\Omega_0$ (dotted orange line).

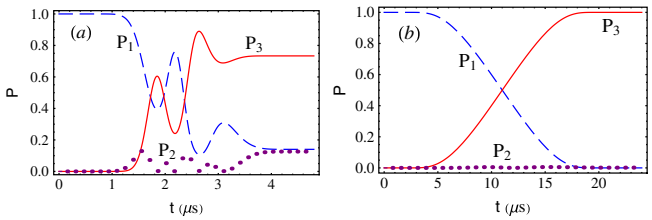

FIG. 4 (color online). Time evolution of the populations of levels 1, 2, and 3 for STIRAP: (a) Hamiltonian $H_0^I(t)$, (b) $\tilde{H}_1^I(t)$. Same parameters as in Fig. 3.

$$\tilde{H}_1^I(t) = \frac{\hbar}{2}\begin{pmatrix} 0 & 0 & i\tilde{\Omega}_a \\ 0 & 0 & 0 \\ -i\tilde{\Omega}_a & 0 & 0 \end{pmatrix}, \tag{6}$$

where $\tilde{\Omega}_a = 2\dot{\theta}$. The simplest shortcut recipe in this case is to substitute the original two-photon transition by a special one-photon 1-3 pulse. As in RAP, the Rabi frequency can be kept real without affecting the results. For alkali atoms one can generate $\tilde{H}_1^I$ with microwaves exploiting the atomic clock transition between ground state hyperfine levels, with 1 and 3 as the nonmagnetic states ($m_F = 0$) of each hyperfine level. At variance with RAP, the physical implementation does not require here any level shift engineering. Figures 3 and 4 show examples where [21]

$$\Omega_p(t) = \Omega_0 f(t-\tau); \qquad \Omega_s(t) = \Omega_0 f(t);$$
$$f(t) = \begin{cases} \sin^4(\pi t/T) & (0 < t < T) \\ 0 & (\text{otherwise}). \end{cases} \tag{7}$$

Figure 3 shows a STIRAP Stokes-pump pulse sequence where adiabaticity breaks down; see Fig. 4(a). Complete transfer can be achieved using the interaction in (6); see Figs. 3 and 4(b). With our parameters the maximum of $\tilde{\Omega}_a/(2\pi)$ is at 159 kHz, which can be reached with available intensities.

Discussion and outlook.—Similar ideas are applicable to speed up adiabatic approaches in quantum information processing [22–25], interferometric methods in superconducting qubits [26] or quantum dots [27], the creation of entangled states [28], and techniques using avoided crossing passage [29], keeping their stability versus parameter variations. The SHAPE method is also compatible with approaches that optimize H_0, such as the quantum brachistochrone [23], since, after optimizing the adiabatic process, it leads to even shorter durations.

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