

**Fast bias inversion of a double well without residual particle excitation**S. Martínez-Garaot,<sup>1,\*</sup> M. Palmero,<sup>1</sup> D. Guéry-Odelin,<sup>2</sup> and J. G. Muga<sup>1,3</sup><sup>1</sup>*Departamento de Química Física, UPV/EHU, Apdo 644, 48080 Bilbao, Spain*<sup>2</sup>*Laboratoire de Collisions Agrégats Réactivité, CNRS UMR 5589, IRSAMC, Université de Toulouse (UPS), 118 Route de Narbonne, 31062 Toulouse CEDEX 4, France*<sup>3</sup>*Department of Physics, Shanghai University, 200444 Shanghai, People's Republic of China*

(Received 14 July 2015; published 5 November 2015)

We design fast bias inversions of an asymmetric double well so that the lowest states in each well stay in the same well they started, free from residual motional excitation. This cannot be done adiabatically, and a sudden bias switch produces in general motional excitation. The residual excitation is suppressed by complementing a predetermined fast bias change with a linear ramp whose time-dependent slope compensates for the displacement of the wells. The process, combined with vibrational multiplexing and demultiplexing, can produce vibrational state inversion without exciting internal states, just by deforming the trap.

DOI: [10.1103/PhysRevA.92.053406](https://doi.org/10.1103/PhysRevA.92.053406)

PACS number(s): 32.80.Qk, 37.10.Gh, 37.10.Vz

**I. INTRODUCTION**

In a recent paper [1] a protocol to realize fast vibrational state multiplexing or demultiplexing of ultracold atoms was introduced. By a properly designed time-dependent potential deformation between a harmonic trap and a biased double well, the states of a single atom in a harmonic trap can be dynamically mapped into states localized at each well (vibrational demultiplexing; see the left arrow in Fig. 1), or vice versa (multiplexing; see the right arrow in Fig. 1), faster than adiabatically and without residual excitation at the final time. It was suggested that these processes may be combined with a bias inversion to produce (i) state inversions, from the ground to the first excited state of the harmonic trap and vice versa, based on trap deformations only (see the evolution of gray and white states in Fig. 1), or (ii) to produce vibrationally excited Fock states from an initial ground state [1]. These are basic operations to implement quantum information processing and fundamental studies. Thus the possibility to perform them without exciting internal atomic states as an intermediate step is of much interest. For trapped ions in particular, this amounts to a species-independent approach based entirely on the charge and electric forces. In contrast  $\pi$ -pulse sequences require specific lasers for each system and a suitable level structure. In general, i.e., both for ions and neutral atoms, a method not using internal-state excitation suppresses decoherence and random kicks due to spontaneous decay. They may be important limiting factors to preserve quantum dynamics with optical transitions [2].

Among the possibilities to avoid decay from an intermediate state in a transition among motional states, one might think of using Stimulated Raman adiabatic passage (STIRAP) [3], which in principle does not populate the upper, intermediate state. This technique, however, is best suited for transitions involving a change in internal state, and its application to purely vibrational transitions (within the same internal state) is not straightforward. Numerical simulations show that several motional states are populated [4], and in fact the experimental applications of STIRAP for trapped ions have been only used for inducing carrier or sideband transitions that involve changing the internal state [5].

The objective of this paper is to design fast controlled bias inversions of a double well so that the lowest states in each well remain as lowest states without residual excitation. Unlike multiplexing, however, there is no truly adiabatic slow process that achieves this state transformation. In the bias inversion depicted within the dashed-line frame of Fig. 1, for example, a slow bias change would preserve the order of the states according to their energy, so that the states represented in the third potential configuration would be interchanged (i.e. the gray state in the right well, and the white one in the left well). Nevertheless, in the limit in which the two wells are effectively independent, which in practice means, for times shorter than the tunneling time among the wells, the intended state transition might indeed be done slowly enough to be considered adiabatic. If we approximate each “isolated” well by a harmonic oscillator, the intended transformation amounts to a “horizontal” displacement along the interwell axis together with a rising or lowering of the energy of the wells. The latter effects, however, do not affect the intrawell dynamics, so we may focus on the displacement. In other words, within the stated approximations the bias inversion amounts to transporting a particle in a harmonic potential. Thus, to achieve a fast transition without residual excitation we may use shortcuts to adiabaticity designed to perform fast transport [6]. Specifically we shall use a compensating-force approach [7,8], equivalent to the fast-forward scaling technique [9], based on adding to the potential a linear ramp with time-dependent slope to compensate for the effect of the trap motion in the noninertial frame of the trap. We shall compare this approach with a sudden bias switch, a fast quasiasiabatic (FAQUAD) approach [10], or a smooth polynomial connection without compensation. In Sec. II we introduce the compensating-force approach and Sec. III describes the alternative methods. In Sec. IV numerical examples are presented with parameters appropriate for trapped ions in multisegmented, linear Paul traps, and for neutral atoms in optical traps. Finally, in Sec. V we discuss the results and open questions.

**II. COMPENSATING-FORCE APPROACH**

If the double-well potential with nearly independent wells is subjected to a bias inversion such that the trap frequencies of each well are essentially equal and constant throughout, and

\*sofia.martinez@ehu.es

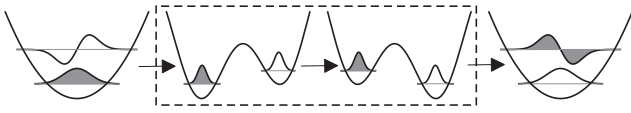


FIG. 1. Schematic representation of demultiplexing (left arrow), bias inversion (framed in dashed line, central arrow), and multiplexing (right arrow). The densities of two one-atom eigenstates are represented in all potentials. In the harmonic potentials (unframed potentials on the left and right charts) the states are the ground state and first excited state. In the two central charts with tilted double wells the states are the lowest for each well. The color (white or gray) indicates how they would evolve sequentially following the fast protocol described in the text. For example, the gray state is initially the ground state of the harmonic oscillator, then it becomes the lowest state of the left well, and remains being the lowest state of that well after the bias inversion. In the last step it becomes the first excited state of the final harmonic oscillator.

the trajectories of the well minima move in parallel, the process may be described by a parallel transport of two rigid harmonic oscillators, one for each well. The Hamiltonian potential near the minima may be approximated as

$$V_0(x - x_0) = \frac{1}{2}m\Omega_0^2(x - x_0)^2, \quad (1)$$

where  $\Omega_0$  is the angular frequency and  $x_0 = x_0(t)$  is the common notation for either of the two minima.<sup>1</sup> When needed we may distinguish the minima as  $x_{0,\pm}$ , with  $x_{0,+} > x_{0,-}$ . The Hamiltonian  $H_0 = p^2/2m + V_0(x - x_0)$  has eigenenergies  $E_n = (n + \frac{1}{2})\hbar\Omega_0$ , and well-known normalized eigenstates  $\phi_n(x - x_0)$ , proportional to Hermite polynomials [11].

Adding to the Hamiltonian a linear term with an appropriate time-dependent slope the noninertial effect of the motion of the well will be compensated in the trap frame [7,8]. To define the trap frame consider the following position and momentum displacement unitary operator

$$\mathcal{U} = e^{ipx_0(t)/\hbar} e^{-im\dot{x}_0(t)x/\hbar}, \quad (2)$$

where the overdot represents a time derivative. Starting from the Schrödinger equation

$$i\hbar\partial_t|\psi\rangle = H_0|\psi\rangle, \quad (3)$$

the transformed wave function  $|\Phi\rangle = \mathcal{U}|\psi\rangle$  obeys

$$i\hbar\partial_t|\Phi\rangle = \mathcal{U}H_0\mathcal{U}^\dagger|\Phi\rangle + i\hbar(\partial_t\mathcal{U})\mathcal{U}^\dagger|\Phi\rangle = H'_0|\Phi\rangle, \quad (4)$$

where the interaction picture (trap frame) Hamiltonian is

$$H'_0 = \frac{p^2}{2m} + V_0(x) + m\dot{x}_0x + \frac{1}{2}m\dot{x}_0^2, \quad (5)$$

and  $V_0(x) = \frac{1}{2}m\Omega_0^2x^2$ . The term  $m\dot{x}_0^2/2$  only depends on time; it does not affect the dynamics and can be ignored. To compensate the motion of the trap, we add  $-m\dot{x}_0x$  to  $H_0$ . This produces  $-m(x + x_0)\dot{x}_0$  in the trap frame and the resulting potential in that frame is reduced to  $V_0(x)$ , again neglecting purely time-dependent functions.  $V_0(x)$  does not depend on

<sup>1</sup>We disregard purely time-dependent functions in each well. Differential phases among the wells depending on these functions can be ignored since the traps are assumed to be independent.

time, so any stationary state in this trap frame will remain stationary, and excitations are avoided.

### III. ALTERNATIVE METHODS

In this section we discuss three simple alternative approaches to perform the bias inversion. They are all quite natural and simple approaches whose performance can be compared to that of the compensating force approach.

*Sudden approach.* In the sudden approach the potential is changed abruptly from the initial to the final configuration, but the state of the system remains unchanged immediately after the potential change (in general it will evolve afterwards). If the target state is  $\psi_{\text{tar}}$  the resulting fidelity is

$$F_s = |\langle\psi(0)|\psi_{\text{tar}}\rangle|. \quad (6)$$

*FAQUAD.* A quasiadiabatic method to speed up adiabatic processes when there is one control parameter  $\lambda(t)$  is based on distributing the adiabaticity parameter homogeneously in time [10]. For instantaneous levels 0 and 1 this means

$$\hbar \left| \frac{\langle\phi_0|\partial_t\phi_1\rangle}{E_0 - E_1} \right| = c, \quad (7)$$

where the instantaneous eigenstates  $\phi_0, \phi_1$  and eigenenergies  $E_0, E_1$  depend on time through their dependence on  $\lambda$ , and  $c$  is constant. By the chain rule this becomes a first order differential equation for  $\lambda(t)$ , and  $c$  is set so that the boundary conditions for  $\lambda(t)$  at initial,  $t = 0$ , and final time  $t_f$  are satisfied. In the transport of a particle with a harmonic oscillator of angular frequency  $\Omega_0$  centered at  $x_0(t)$  we set  $\lambda(t) = x_0(t)$ . Using the energies and eigenstates of the first two levels of the harmonic oscillator in Eq. (7), the FAQUAD condition becomes simply

$$\frac{m\dot{x}_0(t)}{\sqrt{2\hbar m\Omega_0}} = c. \quad (8)$$

The solution is the linear connection  $x_0(t) = x_0(0) + [x_0(t_f) - x_0(0)]t/t_f$ . The minimal  $t_f$  for which a zero of excitation energy appears is [10,12]  $2\pi/\Omega_0$ .

*Polynomial connection without compensation.* The final and initial values of the control parameter may as well be smoothly connected without applying any compensation, for example, using a fifth order polynomial that assures the vanishing of first and second derivatives of the parameter at the boundary times.

### IV. EXAMPLES

In the following examples, the potentials and parameters are adapted for a trapped ion in a multisegmented Paul trap, and for a neutral atom in a dipole trap.

#### A. Trapped ions

For a trapped ion we consider a simple double-well potential of the form

$$V(x, t) = \beta x^4 + \alpha x^2 + \gamma x, \quad (9)$$

with  $\alpha(t) < 0$  and  $\beta(t) > 0$  [13–15].  $\alpha$  and  $\beta$  are assumed to be constant and  $\gamma \equiv \gamma(t)$  time dependent. The bias inversion implies a change of sign of  $\gamma(t)$  from  $\gamma_0 > 0$  to  $-\gamma_0$ .

From  $\frac{\partial V}{\partial x} = 0$  the condition for the extrema becomes

$$4\beta x^3 + 2\alpha x + \gamma = 0. \quad (10)$$

It is useful to define

$$A = 0, \quad B = \frac{2\alpha}{4\beta}, \quad C = \frac{\gamma}{4\beta}, \quad (11)$$

$$Q = \frac{A^2 - 3B}{9}, \quad R = \frac{2A^3 - 9AB + 27C}{54}. \quad (12)$$

When  $R^2 < Q^3$  there are two minima and one maximum. With  $\alpha < 0$  and  $\beta > 0$ , this is satisfied for

$$|\gamma| < \left(\frac{2}{3}\right)^{3/2} \sqrt{-\frac{\alpha^3}{\beta}}. \quad (13)$$

The trajectories of the minima are

$$x_{0,\pm} = -2\sqrt{Q} \cos\left(\frac{\theta + (1 \pm 1)\pi}{3}\right) - \frac{A}{3}, \quad (14)$$

where  $\theta = \arccos\left(\frac{R}{\sqrt{Q^3}}\right)$ ,  $0 \leq \theta \leq \pi$  and the roots are taken to be positive. Up to second order in  $\gamma$  they are

$$x_{0,-} \approx -\frac{1}{\sqrt{2}} \sqrt{-\frac{\alpha}{\beta}} + \frac{\gamma}{4\alpha} - \frac{3\gamma^2 \sqrt{-\alpha\beta}}{16\sqrt{2}\alpha^3}, \quad (15)$$

$$x_{0,+} \approx \frac{1}{\sqrt{2}} \sqrt{-\frac{\alpha}{\beta}} + \frac{\gamma}{4\alpha} + \frac{3\gamma^2 \sqrt{-\alpha\beta}}{16\sqrt{2}\alpha^3}. \quad (16)$$

The quadratic term in  $\gamma$  is negligible with respect to the linear term when

$$|\gamma| \ll \frac{4\sqrt{2}}{3} \sqrt{-\frac{\alpha^3}{\beta}}, \quad (17)$$

which implies that the trajectories for the minima move in parallel. Note that this inequality automatically implies the one in Eq. (13). Neglecting the quadratic term, the two minima are given by  $x_{0,\pm} = \pm \frac{1}{\sqrt{2}} \sqrt{-\frac{\alpha}{\beta}} + \frac{\gamma}{4\alpha}$ . The distance between the minima is

$$D = 2\sqrt{Q} \left\{ \cos\left(\frac{\theta}{3}\right) + \sin\left[\frac{1}{6}(\pi + 2\theta)\right] \right\} \\ \approx \sqrt{2} \sqrt{-\frac{\alpha}{\beta}} + \frac{3\sqrt{-\alpha\beta}}{8\sqrt{2}\alpha^3} \gamma^2. \quad (18)$$

We may also compute the energy bias between the two wells as

$$\delta = \gamma D. \quad (19)$$

The distance traveled by each well is, when (17) is fulfilled,  $d = \gamma_0/(2\alpha)$  [see Eqs. (15) and (16)], and the effective frequency at each minimum

$$\omega_0 = \sqrt{\frac{1}{m} \left( \frac{d^2 V}{dx^2} \right)_{x=x_0}}. \quad (20)$$

For Eq. (9) the effective frequencies are

$$\omega_{0,\pm} = \sqrt{\frac{2}{m} \sqrt{\alpha + \frac{2}{3}\beta} \left\{ A + 6\sqrt{Q} \cos\left[\frac{1}{3} \left( \frac{\theta + (1 \pm 1)\pi}{3} \right) \right] \right\}^2} \\ \approx 2\sqrt{-\frac{\alpha}{m}} \mp \frac{3}{2\sqrt{2}} \sqrt{\frac{\beta}{\alpha^2 m}} \gamma. \quad (21)$$

Hence, comparing the two terms, the condition for the frequencies to be essentially constant  $\omega_{0,\mp} \approx \Omega_0 \equiv 2\sqrt{-\frac{\alpha}{m}}$  is again the inequality in Eq. (17).

In the regime where the inequality (17) holds, we can apply the compensating force approach to implement a fast bias inversion. Since the compensating term  $-m\ddot{x}_0$  is equal for both harmonic traps, we add it to  $V$  in Eq. (9), and the resulting Hamiltonian  $H$  is

$$H = \frac{p^2}{2m} + \beta x^4 + \alpha x^2 + (\gamma - m\ddot{x}_0)x. \quad (22)$$

Note that the compensation amounts to changing the time dependence of the slope of the linear term from the reference process defined by  $\gamma(t)$  to  $\gamma_{\text{eff}}(t) \equiv \gamma(t) - m\ddot{x}_0(t) = \gamma(t) - m\dot{\gamma}(t)/(4\alpha)$ . To set  $\gamma(t)$  we design a connection between the initial and final configurations. First note the boundary conditions

$$\gamma(0) = \gamma_0 > 0, \quad \gamma(t_f) = -\gamma_0, \quad (23)$$

which we complement by

$$\dot{\gamma}(t_b) = 0, \quad \ddot{\gamma}(t_b) = 0, \quad t_b = 0, t_f, \quad (24)$$

so that  $\dot{x}_0(t_b) = \ddot{x}_0(t_b) = 0$ . This implies that  $\mathcal{U}(t_b) = e^{ipx_0(t_b)/\hbar}$  and  $\dot{\mathcal{U}}(t_b) = 0$ . Therefore the Hamiltonians and the wave functions in interaction and Schrödinger pictures transform into each other by a simple coordinate displacement. At intermediate times, we interpolate the function as  $\gamma(t) = \sum_{n=0}^5 c_n t^n$ , where the coefficients are found by solving Eqs. (23) and (24). Finally,

$$\gamma(t) = \gamma(0) + 10[\gamma(t_f) - \gamma(0)]s^3 \\ - 15[\gamma(t_f) - \gamma(0)]s^4 + 6[\gamma(t_f) - \gamma(0)]s^5, \quad (25)$$

where  $s = t/t_f$ . This function and examples of  $\gamma_{\text{eff}}$  are shown in Fig. 2.

In order to compare the robustness of the compensating force method against the alternative ones we consider a  ${}^9\text{Be}^+$  ion in the double well with the realistic parameters  $\alpha = -4.7$  pN/m and  $\beta = 5.2$  mN/m<sup>3</sup>, similar to those in [16]. For a moderate initial bias compared to the vibrational quanta, such as

$$\gamma_0 \sim \frac{\hbar\Omega_0}{D}, \quad (26)$$

the fidelity provided by the sudden approach is one for all practical purposes so we can change the bias abruptly and reach the target state. The displacement of the trap  $d$  may be compared with the oscillator characteristic length  $a_0 = \sqrt{\hbar/m\Omega_0}$ . Their ratio is

$$R = \frac{d}{a_0} = \frac{\gamma_0}{2\alpha} \sqrt{\frac{m\Omega_0}{\hbar}}. \quad (27)$$

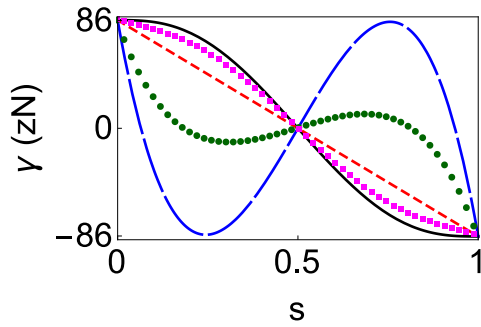


FIG. 2. (Color online)  $\gamma$  versus  $s = t/t_f$  for the polynomial in Eq. (25) (solid black line) and FAQUAD (short-dashed red line).  $\gamma_0 = 86.4$  zN,  $\gamma(t_f) = -\gamma_0$ ,  $\alpha = -4.7$  pN/m, and  $\beta = 5.2$  mN/m<sup>3</sup>. Also shown are the different effective slopes adding a compensation to the polynomial,  $\gamma_{\text{eff}}(t) = \gamma(t) - m\ddot{\gamma}(t)/(4\alpha)$ , for the mass of <sup>9</sup>Be<sup>+</sup> and times  $t_f = 0.07$   $\mu\text{s}$  (long-dashed blue line),  $t_f = 0.1$   $\mu\text{s}$  (green dots), and  $t_f = 0.3$   $\mu\text{s}$  (magenta squares).

For the Paul trap  $R \approx 0.00065$ , which explains the high fidelity of the sudden approach for a moderate bias inversion of the ion. At these bias values there is really no need to apply a more sophisticated protocol than the sudden switch.

Henceforth, we assume a much larger  $\gamma_0$ , but still satisfying the condition (17). In particular, for an initial bias of  $1000 \Omega_0 \hbar$  (corresponding to  $\gamma_0 = 86.4$  zN), the ratio becomes  $R \approx 0.65$ . The maximum variation of the difference between the trajectories of the minima is 3 pm, about three orders of magnitude less than the displacement of each minimum (9.2 nm), so the minima follow parallel trajectories. Furthermore, the maximum variation of the frequencies in Eq. (21) with respect to  $\Omega_0 = 2\pi \times 5.6$  MHz is  $2\pi \times 3.7$  kHz, so the effective frequency is nearly constant.

Figure 3 demonstrates the effect of the compensating-force approach. Starting from the ground state of the lower (left) well, the final evolved state following the shortcut with compensation stays as the “ground state” of the left well. This is actually defined as the lowest state of the double-well system predominantly located on the left. There is a similar process for the right well. The final states represented are obtained by solving the Schrödinger equation with the full Hamiltonian (22).

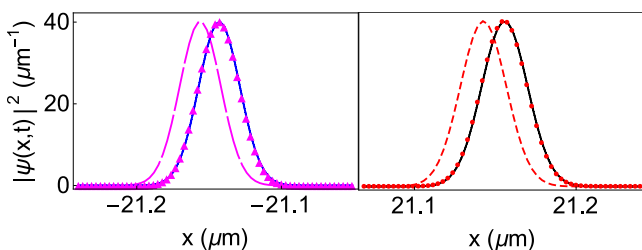


FIG. 3. (Color online) Left: Ground state of the left well at  $t = 0$  (long-dashed magenta line) and at  $t = t_f$  (magenta triangles), and final state with the compensating force applied on the double well (solid blue line). Right: Ground state of the right well: at  $t = 0$  (short-dashed red line) and at  $t = t_f$  (red dots) and final state with the compensating force applied (solid black line).  $t_f = 4$  ns and other parameters as in Fig. 2 for <sup>9</sup>Be<sup>+</sup>.

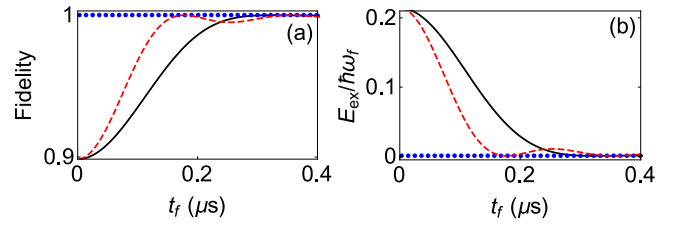


FIG. 4. (Color online) (a) Fidelity  $|\langle \phi_L(t_f) | \psi(t_f) \rangle|$ , where  $|\phi_L(t_f)\rangle$  is the lowest state located in the left well in the final time configuration, and  $|\psi(t_f)\rangle$  is the evolved state following the shortcut at final time. (b) Final excitation energy for the process on the left well using compensating force (blue dots), fifth degree polynomial in Eq. (25) (solid black line), and FAQUAD (short-dashed red line). The parameters are for <sup>9</sup>Be<sup>+</sup> as in Fig. 2.

Figure 4 is for the process in the left well. It compares the fidelity at final time and the excitation energy, defined as  $E_{\text{ex}} = E(t_f) - E_0(t_f)$  where  $E(t_f)$  is the final energy after the quantum evolution following the shortcut and  $E_0(t_f)$  is the ground state final energy of the upper harmonic well, using the polynomial (25) for  $\gamma$  with and without compensation, as well as the results of the FAQUAD approach. The fidelity without compensation tends to the fidelity of the sudden approach (0.89) for very short final times. The method with compensation clearly outperforms the others. In principle a fundamental limitation of the approach is due to the fact that the inequality (17), which guarantees parallel motion and stable frequencies of the wells, should as well be satisfied by  $\gamma_{\text{eff}}$ , but, at very short times, the dominant term of  $\gamma_{\text{eff}} \sim -m\ddot{\gamma}/4\alpha$  may be too large. To estimate this short time limit we combine the mean-value theorem inequality for the maximum [7],  $|\ddot{\gamma}|_{\text{max}} \geq 4\gamma_0/t_f^2$ , with Eq. (17) for  $\gamma_{\text{eff}}$  to find the condition

$$t_f \gg \left( \frac{3m\gamma_0}{4\sqrt{2}} \sqrt{\frac{\beta}{\alpha^5}} \right)^{1/2}. \quad (28)$$

The factor on the right-hand side is  $10^{-9}$  s for this example (see Fig. 4), which is several orders of magnitude smaller than  $2\pi/\Omega_0$  and does not affect the fidelity in the scale of times shown.

## B. Neutral atoms

The potential

$$V_{na}(x,t) = \frac{1}{2}m\omega^2x^2 + V_0 \cos^2 \left[ \frac{\pi(x - \Delta x)}{d_l} \right] \quad (29)$$

forms also a double well. It was implemented in [17] with optical dipole potentials, combining a harmonic confinement due to a crossed beam dipole trap with a periodic light shift potential provided by the interference pattern of two mutually coherent laser beams.  $\omega$  is the angular frequency of the dipole trap about the waist position,  $V_0$  the amplitude,  $\Delta x$  the displacement of the optical lattice relative to the center of the harmonic well, and  $d_l$  is the lattice constant. (Double wells with a controllable bias may be also realized by two optical lattices of different periodicity with controllable intensities and relative phase [18]). Here, the bias inversion implies a change of sign of  $\Delta x(t)$  from  $\Delta x_0 > 0$  to  $-\Delta x_0$ .



To check if the conditions to apply the compensating force approach hold here as well, an analysis similar to the one in the previous example is now performed. We approximate the potential around each minimum,  $V^{(\pm)}$ , up to fourth order. From  $\frac{\partial V^{(\pm)}}{\partial x} = 0$  we get a cubic equation for each minimum. The positions of the minima are thus given by

$$x_{0,\pm} = -2\sqrt{Q} \cos\left(\frac{\theta^{(\pm)} - 2\pi}{3}\right) - \frac{A^{(\pm)}}{3}, \quad (30)$$

where

$$Q = \frac{2d_l^2\pi^2V_0 + d_l^4m\omega^2}{4\pi^4V_0},$$

$$A^{(\pm)} = -\frac{3}{2}(2\Delta x \pm d_l), \quad (31)$$

$$\theta^{(\pm)} = \cos\left[\frac{-3d_l(2\Delta x \pm d_l)m\pi^2\sqrt{V_0\omega^2}}{2(2\pi^2V_0 + d_l^2m\omega^2)^{3/2}}\right]^{-1}.$$

Up to a second order in  $\Delta x$ ,

$$x_{0,\pm} \approx \pm a + b\Delta x \pm c\Delta x^2, \quad (32)$$

where the coefficients are known explicitly but too lengthy to be displayed here. Whenever the quadratic term is negligible with respect to the linear term ( $c\Delta x^2 \ll b\Delta x$ ), we can approximate  $x_{0,\pm} = \pm a + b\Delta x$  (parallel motion). The distance between the minima is given by

$$D = \frac{1}{3} \left\{ A^{(-)} - A^{(+)} + 6\sqrt{Q} \left[ -\cos\left(\frac{\pi + \theta^{(-)}}{3}\right) + \cos\left(\frac{\pi + \theta^{(+)}}{3}\right) \right] \right\} \approx 2a + 2c\Delta x^2. \quad (33)$$

Moreover,  $\omega_{0,\pm} \approx f \mp g\Delta x$ , again with known but lengthy coefficients  $g$  and  $f$ . As long as  $g\Delta x \ll f$  we may set  $\omega_{0,\pm} \approx \Omega_0 \equiv f$ .

For realistic parameters the conditions for parallel transport of the minima and constant frequency are indeed satisfied. We consider a  $^{87}\text{Rb}$  atom in the trap and set the parameters in [1] after the demultiplexing process, namely,  $d_l = 5.18 \mu\text{m}$ ,  $\omega = 2\pi \times 59.4 \text{ Hz}$ , and  $V_0/h = 1.4 \text{ kHz}$ ; the time-dependent displacement  $\Delta x = \Delta x(t)$  is the control parameter to perform the bias inversion, such that

$$\Delta x(0) = \Delta x_0, \quad \Delta x(t_f) = -\Delta x_0, \quad (34)$$

with  $\Delta x_0 = 200 \text{ nm}$ . We also impose

$$\begin{aligned} \dot{\Delta x}(0) &= 0, & \ddot{\Delta x}(0) &= 0, \\ \dot{\Delta x}(t_f) &= 0, & \ddot{\Delta x}(t_f) &= 0, \end{aligned} \quad (35)$$

to achieve similar conditions in the derivatives of the minima  $x_0$ . At intermediate times, we interpolate the function as  $\Delta x(t) = \sum_{n=0}^5 d_n t^n$ , where the coefficients are found by solving Eqs. (34) and (35). Consequently, the connection between the initial and final Hamiltonians is given by the same polynomial in Eq. (25) changing  $\gamma(t) \rightarrow \Delta x(t)$ . The double wells are much deeper and tight for trapped ions than for neutral atoms; compare an intrawell angular frequency  $\Omega_0$  of  $2\pi \times 5.6 \text{ MHz}$  for the ions versus  $2\pi \times 0.35 \text{ kHz}$  for the optical trap. Therefore, in this case, for a moderate initial

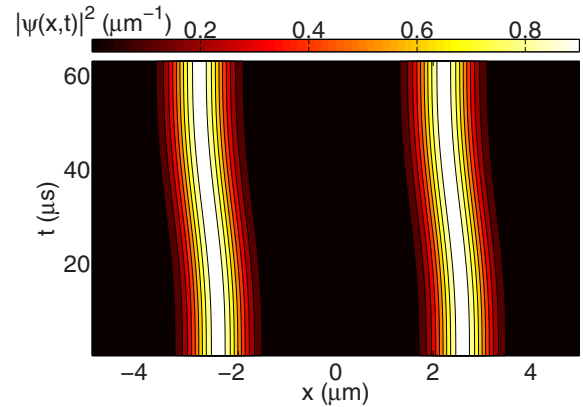


FIG. 5. (Color online) Evolution of the wave function densities following the shortcut in Eq. (36) for states in left and right wells. The parameters are for  $^{87}\text{Rb}$ :  $d_l = 5.18 \mu\text{m}$ ,  $\omega = 59.4 \times 2\pi \text{ Hz}$ ,  $V_0/h = 1.4 \text{ kHz}$ ,  $\Delta x_0 = 200 \text{ nm}$ , and  $t_f = 63 \mu\text{s}$ .

bias compared to the vibrational quanta, the ratio between the displacement of the trap  $d$  and the oscillator characteristic length  $a_0$  is  $R \approx 0.67$ .

With the parameters given at time  $t = 0$ , the separation of the minima is  $D = 5 \mu\text{m}$ , the bias between minima  $\delta = 2.02 \times 10^{-32} \text{ J}$ , and an the effective angular frequency  $\Omega_0 = 2\pi \times 0.35 \text{ kHz}$ , whereas the maximum variation of the frequencies along the process is approximately  $2\pi \times 0.2 \text{ Hz}$ . Furthermore, the maximum deviation from  $D$  of the minima separation is  $1.8 \text{ nm}$ , whereas the displacement of each minimum is about  $0.4 \mu\text{m}$ . In summary, the minima do move in parallel with constant, equal frequencies for practical purposes.

To accelerate the bias inversion we add the compensating term to  $V$  in Eq. (29),

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 + V_0 \cos^2\left[\frac{\pi(x - \Delta x)}{d_l}\right] - mx\ddot{x}_0. \quad (36)$$

Figure 5 shows the evolution of the densities. Focusing on the left well, Fig. 6(a) demonstrates that the fidelity is exactly one (blue dots) with the compensating force. However, using for the inversion the fifth degree polynomial in Eq. (25) [with

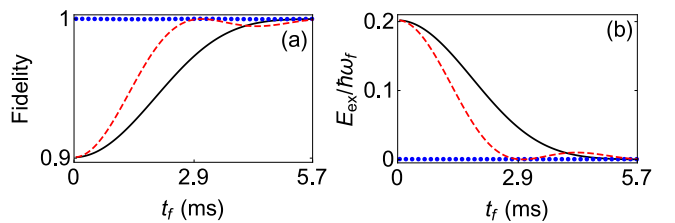


FIG. 6. (Color online) (a) Fidelity  $|\langle\varphi_1(t_f)|\psi(t_f)\rangle|$ , where  $|\varphi_1(t_f)\rangle$  is the lowest state predominantly of the left well at final time (the first excited state of the double well) and  $|\psi(t_f)\rangle$  is the evolved state following the shortcut at final time. (b) Final excitation energy. Compensating-force approach (blue dots), fifth degree polynomial in Eq. (25) with the change  $\gamma(t) \rightarrow \Delta x(t)$  without compensation (solid black line), and FAQUAD approach (short-dashed red line). The parameters are chosen for  $^{87}\text{Rb}$ :  $d_l = 5.18 \mu\text{m}$ ,  $\omega = 59.4 \times 2\pi \text{ Hz}$ ,  $V_0/h = 1.4 \text{ kHz}$ , and  $\Delta x_0 = 200 \text{ nm}$ .

the change  $\gamma(t) \rightarrow \Delta x(t)$ ] without compensation the fidelity at short final times decreases until the value of the sudden approach, 0.9. Furthermore, the excitation (residual) energy  $E_{\text{ex}}$  is approximately zero for the shortcut protocol, compared to the excitation without compensation in Fig. 6(b).

## V. DISCUSSION

In this work we have proposed a method to invert the bias of a double-well potential, in the regime of independent wells, to keep the final states motionally unexcited within the same original well. The method treats the bias inversion as a rigid transport of the wells, which is justified for realistic parameters, and applies a “compensating force” to cancel the excitations. Examples have been worked out for ions or neutral atoms and comparisons have been provided with a sudden approach, a fast quasiadiabatic (FAQUAD) approach, or a smooth polynomial connection of initial and final bias without compensation. The compensating-force method clearly outperforms the others and gives perfect fidelities under ideal conditions, up to very small times compared, e.g., with the time  $2\pi/\Omega_0$  (one oscillation period) where FAQUAD provides a first zero of the excitation energy. The feasibility of the approach may be analyzed in the light of current technology in the two systems examined:

(1) For trapped ions we have considered initial and final values differing by  $\Delta\gamma \approx 200$  zN, whereas resolutions of 15 zN have been reported [19]. As for the timing, much effort is being put into achieving suboscillation-period resolutions for the potential update [20–22] in ion transport experiments. The possibility to switch on and off potentials in a few nanoseconds, much faster than the ion oscillation period, has been demonstrated [22]. The designed bias inversion is thus in reach with current technology.

(2) For neutral atoms, the minimal process times  $t_f$  are not limited by the method per se but by the technical capabilities to implement the maximal compensating force. This force depends on the maximal acceleration of the well, whose lower bound is known to be  $a_{\text{max}} = 2d/t_f^2$  [7]. To implement the compensation with a magnetic field gradient  $G$ , the gradient should be of the order of  $G \simeq ma_{\text{max}}/\mu_B$  in an amount of time  $t_f$  ( $\mu_B$  is Bohr’s magneton). For rubidium atoms polarized in

the magnetic level  $F = m_F = 2$  and the transport parameters considered above, this requires a magnetic field gradient on the order of 40 T/m shaped on a time interval  $t_f = 63 \mu\text{s}$ . This is definitely challenging from an experimental point of view. Alternatively, one can use the dipole force of an out of axis Gaussian laser beam. If the double well is placed on the edge at a distance of  $w/2$  where  $w$  is the waist and if  $\alpha_p$  denotes the polarizability, the local potential slope experienced by the atoms is on the order of  $\alpha_p P/w^3$  where  $P$  is the power of the beam. The compensation requires that  $P/w^3 = m/\alpha_p$ . For instance, with an out-of-resonance beam at a wavelength of 1  $\mu\text{m}$ , the polarizability of rubidium-87 atoms is  $\alpha_p \simeq 1.3 \times 10^{-36} \text{ m}^2 \text{ s}$ , and the compensation can be performed using a 1 W laser with a waist of 20  $\mu\text{m}$ . As for the timing, a submicrosecond time scale for shaping the offset potential is perfectly achievable using a control of the intensity based on acousto-optics modulators.

A relevant feature of the proposed approach is that the reference process used to design the corresponding compensation (we have used a polynomial for simplicity) may be chosen among a broad family of functions satisfying Eqs. (23) and (24). As in other shortcut-to-adiabaticity approaches, this flexibility may be used to enhance robustness versus noise and perturbations [23–25].

The bias inversion put forward here and the multiplexing and demultiplexing protocols developed in [1] provide the necessary toolkit to perform vibrational state inversions [26,27], or Fock state preparations [1]. Applications in optical waveguide design are also feasible [28]. As well, the fast bias inversion is directly applicable to Bose-Einstein condensates [29,30]. Generalizations for conditions in which rigid transport does not hold are also possible using invariant theory [7], which allows for finding processes without final excitation where both the frequency and position of the well depend on time [31].

## ACKNOWLEDGMENTS

We acknowledge U. Poschinger, D. Leibfried, P. O. Schmidt, and J. Alonso for useful comments. This work was supported by the Basque Country Government (Grant No. IT472-10), Ministerio de Economía y Competitividad (Grant No. FIS2012-36673-C03-01), and the program UFI 11/55. S.M.-G. and M.P. acknowledge fellowships by UPV/EHU.

- 
- [1] S. Martínez-Garaot, E. Torrontegui, X. Chen, M. Modugno, D. Guéry-Odelin, S.-Y. Tseng, and J. G. Muga, *Phys. Rev. Lett.* **111**, 213001 (2013).
- [2] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, *Rev. Mod. Phys.* **75**, 281 (2003).
- [3] K. Bergmann, N. V. Vitanov, and B. W. Shore, *J. Chem. Phys.* **142**, 170901 (2015).
- [4] I. Storgaard Vogeliuss, Ph.D. thesis, University of Aarhus, Denmark, 2005.
- [5] F. Gebert, Y. Wan, F. Wolf, Jan C. Heip, and Piet O. Schmidt, [arXiv:1510.00063v1](https://arxiv.org/abs/1510.00063v1).
- [6] E. Torrontegui, S. Ibáñez, S. Martínez-Garaot, M. Modugno, A. del Campo, D. Guéry-Odelin, A. Ruschhaupt, X. Chen, and J. G. Muga, *Adv. At., Mol., Opt. Phys.* **62**, 117 (2013).
- [7] E. Torrontegui, S. Ibáñez, X. Chen, A. Ruschhaupt, D. Guéry-Odelin, and J. G. Muga, *Phys. Rev. A* **83**, 013415 (2011).
- [8] M. Palmero, E. Torrontegui, D. Guéry-Odelin, and J. G. Muga, *Phys. Rev. A* **88**, 053423 (2013).
- [9] S. Masuda and K. Nakamura, *Proc. R. Soc. London, Ser. A* **466**, 1135 (2010).
- [10] S. Martínez-Garaot, A. Ruschhaupt, J. Gillet, Th. Busch, and J. G. Muga, *Phys. Rev. A* **92**, 043406 (2015).
- [11] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill, New York, 1981).
- [12] R. Bowler, J. Gaebler, Y. Lin, T. R. Tan, D. Hanneke, J. D. Jost, J. P. Home, D. Leibfried, and D. J. Wineland, *Phys. Rev. Lett.* **109**, 080502 (2012).

- [13] J. P. Home and A. M. Steane, *Quantum Inf. Comput.* **6**, 289 (2006).
- [14] A. H. Nizamani and W. K. Hensinger, *Appl. Phys. B* **106**, 327 (2012).
- [15] H. Kaufmann, T. Ruster, C. T. Schmiegelow, F. Schmidt-Kaler, and U. G. Poschinger, *New J. Phys.* **16**, 073012 (2014).
- [16] A. C. Wilson, Y. Colombe, K. R. Brown, E. Knill, D. Leibfried, and D. J. Wineland, *Nature (London)* **512**, 57 (2014).
- [17] R. Gati, M. Albiez, J. Fölling, B. Hemmerling, and M. K. Oberthaler, *Appl. Phys. B* **82**, 207 (2006).
- [18] S. Fölling, S. Trotzky, P. Cheinet, M. Feld, R. Saers, A. Widera, T. Müller, and I. Bloch, *Nature (London)* **448**, 1029 (2007).
- [19] T. Ruster, C. Warschburger, H. Kaufmann, C. T. Schmiegelow, A. Walther, M. Hettrich, A. Pfister, V. Kaushal, F. Schmidt-Kaler, and U. G. Poschinger, *Phys. Rev. A* **90**, 033410 (2014).
- [20] R. Bowler, Ph.D. thesis, University of Colorado, 2015.
- [21] J. Alonso, F. M. Leupold, B. C. Keitch, and J. P. Home, *New J. Phys.* **15**, 023001 (2013).
- [22] J. Alonso, F. M. Leupold, Z. U. Soler, M. Fadel, M. Marinelli, B. C. Keitch, V. Negnevitsky, and J. P. Home, [arXiv:1509.06157](https://arxiv.org/abs/1509.06157).
- [23] A. Ruschhaupt, X. Chen, D. Alonso, and J. G. Muga, *New J. Phys.* **14**, 093040 (2012).
- [24] X.-J. Lu, J. G. Muga, X. Chen, U. G. Poschinger, F. Schmidt-Kaler, and A. Ruschhaupt, *Phys. Rev. A* **89**, 063414 (2014).
- [25] D. Guéry-Odelin and J. G. Muga, *Phys. Rev. A* **90**, 063425 (2014).
- [26] R. Bücker, T. Berrada, S. van Frank, J.-F. Schaff, T. Schumm, J. Schmiedmayer, G. Jäger, J. Grond, and U. Hohenester, *J. Phys. B* **46**, 104012 (2013).
- [27] R. Bücker, J. Grond, S. Manz, T. Berrada, T. Betz, C. Koller, U. Hohenester, T. Schumm, A. Perrin, and Jörg Schmiedmayer, *Nat. Phys.* **7**, 608 (2011).
- [28] S.-Y. Tseng and X. Chen, *Opt. Lett.* **37**, 5118 (2012).
- [29] S. Masuda, *Phys. Rev. A* **86**, 063624 (2012).
- [30] E. Torrontegui, X. Chen, M. Modugno, S. Schmidt, A. Ruschhaupt, and J. G. Muga, *New J. Phys.* **14**, 013031 (2012).
- [31] M. Palmero and J. G. Muga (unpublished).