Shortcut to Adiabatic Passage in Two- and Three-Level Atoms

Xi Chen,1,2 I. Lizuain,1 A. Ruschhaupt,3 D. Guéry-Odelin,4 and J. G. Muga1

1Departamento de Química-Física, UPV-EHU, Apartado 644, 48080 Bilbao, Spain
2Department of Physics, Shanghai University, 200444 Shanghai, People’s Republic of China
3Institut für Theoretische Physik, Leibniz Universität Hannover, Appelstraße 2, 30167 Hannover, Germany
4Laboratoire Collisions Agrégats Réactivité, CNRS UMR 5589, IRSAMC, Université Paul Sabatier, 118 Route de Narbonne, 31062 Toulouse Cedex 4, France

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We propose a method to speed up adiabatic passage techniques in two-level and three-level atoms extending to the short-time domain their robustness with respect to parameter variations. It supplements or substitutes the standard laser beam setups with auxiliary pulses that steer the system along the adiabatic path. Compared to other strategies, such as composite pulses or the original adiabatic techniques, it provides a fast and robust approach to population control.

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Introduction.—Two major routes for manipulating the state of a quantum system with interacting fields are the use of (i) resonant pulses and (ii) adiabatic methods such as “rapid” adiabatic passage (RAP), stimulated Raman adiabatic passage (STIRAP), and their variants. In general terms, simple fixed-area resonant pulses may be fast if intense enough, but unstable with respect to errors or fluctuations of the parameters, whereas adiabatic passage is robust but slow. For many applications, from NMR to quantum information processing, the ideal method should be fast and robust, combining the best of the two worlds. These two requirements are particularly demanding if quantum computing is to become feasible at all. Pulse sequences may be more stable than single pulses, but their use is limited by the longer times required, the need to control phase angles and pulse durations accurately, or off-resonant excitations due to sharp pulse edges [1]. Moreover, the error compensating properties of square-pulse sequences are not preserved when substituting them with smooth pulses so that the design of good sequences requires “a good portion of experience and magic” [2]. In NMR, composite pulses are being superseded by adiabatic passage methods [1], which have also been very successful in chemical reaction dynamics [3], laser cooling, atom optics [4], metrology [5], interferometry [6], or cavity quantum electrodynamics [7]. When robustness is the primary concern, they are quite sufficient, and have as well become basic operations for quantum information processing, to design robust gates [8,9], or in quantum adiabatic computing [10,11]. If speed is also important, however, the limitations may be severe [11].

Given the difficulties of composite pulses, it is natural to look for robustness and high operation rates taking instead the adiabatic methods as the starting point and shortening their duration somehow. We propose here a shortcut to adiabatic passage (“SHAPE” hereafter) using the “transitionless quantum driving” algorithm by Berry [12]. The specific applications we shall discuss are speeded-up versions of (two-level) RAP and (three-level) STIRAP, as canonical examples of other adiabatic methods. Variants such as fractional RAP or STIRAP, and multilevel schemes, may be treated similarly.

The transitionless quantum driving algorithm [12] provides Hamiltonians H(t) for which the adiabatic approximation for the time-dependent wave function evolving with a reference Hamiltonian H0(t) becomes exact. The simplest Hamiltonian, H1(t), steers the dynamics along the instantaneous eigenstates |λ(t)⟩ of H0(t) without transitions among them and without phase factors, formally in an arbitrarily short time,

\[ H_1(t) = i\hbar \sum_n |\partial_t \lambda_n⟩(\lambda_n|. \tag{1} \]

As far as populations are concerned, the addition of H0 is possible, but not necessary, affecting only the phases. Thus H1 may supplement H0, when \( H = H_0 + H_1 \), or substitute it, when \( H = H_1 \). At variance with Lyapunov-control methods [13], \( \dot{H} \) is independent of the time-dependent state so it leads to simpler, linear dynamics, and provides systematically exact solutions for adiabatic following without the need for a trial and error approach [13]. The price to pay is that information on the instantaneous eigenstates is required, but this is not problematic in many cases. The physical realizability of H1 needs a separate study in each system. For example, when H0 describes a particle in a time-dependent harmonic potential, H1 becomes a non-local interaction [14]. (For local-interaction solutions, see [15–17].) For a particle with spin in a time-dependent magnetic field, H1 becomes a time-dependent magnetic field [12]. For the atomic two- and three-level systems studied here, H1 will involve auxiliary laser or microwave interactions. The optional addition of H0 will imply different physical implementations.
Rapid adiabatic passage.—Let us consider first the speeding up of a standard rapid adiabatic passage that inverts the population of two levels, \([1, 2]\), by sweeping the radiation through resonance. This broad spread technique originated in NMR [18] but is used in virtually all fields where two-level systems may be controlled by external interactions, such as laser chemistry, modern quantum optics, or quantum information. When the frequency sweep is shorter than the lifetime for spontaneous emission and other relaxation times, it is termed rapid adiabatic passage.

\(H_0, H_1, \) or \(\dot{H}\) will be formulated in a (quasi-)interaction picture here so they will carry an \(I\) superscript, to distinguish them from Schrödinger picture Hamiltonians (with \(S\) subscript). This will also apply to wave functions. To set \(H_0\) let us assume a semiclassical description of the (electric dipole) interaction with the electric field \(E\). For alkali atoms the Rabi frequency is given by \(\Omega_R = (2\sigma_1 e r L) \cdot E / \hbar\), \(r\) being the position vector of the valence electron. Using the rotating wave approximation (RWA), the Hamiltonian for a laser interaction with linear polarization in \(x\) direction is \(H = \frac{\hbar}{2} [|2\rangle \langle 1| \Omega_R e^{-i\omega t} + |1\rangle \langle 2| \Omega_R e^{i\omega t} + \omega_0 |2\rangle \langle 2| - |1\rangle \langle 1|]]\), with \(\Omega_R\) real here and hereafter. In a laser-adapted quasi-interaction picture based on \(h_0(t) = \frac{h}{2} \frac{\hbar}{2} [|2\rangle \langle 2| - |1\rangle \langle 1|]](t)\), the dynamics of the wave function \(\psi(t) = e^{i\hbar / \hbar \theta} \phi(t)\) is governed by the Hamiltonian \(H_0 = e^{i\hbar / \hbar \theta} H(\hat{H}_0 = e^{-i\hbar / \hbar \theta} H).\) Thanks to the commutation between \(H_0\) and \(H\), a simpler and exact quasi-interaction picture, thanks to the commutation between \(h_0\) and \(h_0\). Using \(|1\rangle = (1, 0, 0, t), |2\rangle = (0, 1, 0, 0), H_0(t) = \frac{\hbar}{2} \left( -\Delta(t) \Omega_R(t) \right), \) where \(\Delta(t) = \omega_0 - \omega_L - i \omega_L\) is the effective detuning, controlled by a change in the carrier frequency or an alteration of the Bohr frequency by Zeeman or Stark shifts.

Note the inverse relation, \(\Delta(t) = \omega_0 - \omega_L - i \omega_L\) is the effective detuning, controlled by a change in the carrier frequency or an alteration of the Bohr frequency by Zeeman or Stark shifts. The instantaneous eigenvectors are \(|\lambda_-(t)\rangle = \cos[\theta(t)/2]|2\rangle - \sin[\theta(t)/2]|1\rangle\) and \(|\lambda_+(t)\rangle = \sin[\theta(t)/2]|2\rangle + \cos[\theta(t)/2]|1\rangle\), with the mixing angle \(\theta(t) \equiv \arccos([-\Delta(t) / \Omega_R(t)]\) and eigenvalues \(E_\pm(t) = \mp \hbar \Omega_R / 2\), where \(\Omega_R = \sqrt{\Delta^2(t) + \Omega_R^2(t)}\). If the adiabatic condition \(\frac{1}{2} |\Omega_R| \ll |\Omega_R|), \Omega_R(t) \Delta(t) / \Omega_R^2(t)\), is satisfied, the state evolving from \(|\psi(t)\rangle = |\lambda_-(t)\rangle\) follows the adiabatic approximation \(|\psi_\pm(t)\rangle = \exp[-i \int \sigma(t) dt] |\lambda_\pm(t)\rangle\), whereas transitions will occur otherwise. Different adiabatic passage schemes correspond to \(\Omega_R(t)\) and \(\Delta(t)\) for which \(\psi_\pm\) goes from one bare state to the other. The simplest one is the Landau-Zener scheme with constant \(\Omega_R\) and linear-in-time \(\Delta\). For the examples below we shall use the more adiabatic (and potentially faster) Allen-Eberly (AE) scheme [19,20]: \(\Omega_R(t) = \Omega_0 \sec(\pi \tau / 2 \omega_0), \quad \Delta(t) = (2 \beta^2 \omega_0 / \pi) \tanh(\pi t / 2 \omega_0).\) Regardless of the scheme chosen, \(H_1(t)\) here takes the form, using Eq. (1),

\[
H_1(t) = \frac{\hbar}{2} \left( 0 -i \Omega_\alpha \right)
\]

where (up to a phase factor) \(\Omega_\alpha\) plays the role of the Rabi frequency for a fast-driving field. In principle, \(H_1\) drives the dynamics along the \(H_1\)-adiabatic path in arbitrarily short times, but there are practical limitations, such as the laser power available. Moreover, a comparison with \(H_0\) dynamics is only fair if \(|\Omega_\alpha|\) is smaller or approximately equal to the peak Rabi frequency in the original laser setup. Independently of the scheme chosen and in a range of interaction times that break down the adiabaticity condition, it is remarkable that the dynamics can be driven along the \(H_0\)-adiabatic path while fulfilling the inequalities \(|\Omega_\alpha| \leq |\Omega_\alpha| \leq |\Omega_0|\).

The physical meaning and realizability of the fast-driving term are determined by going back to the Schrödinger picture: For \(H = H_0 + H_1, \) then \(H = \frac{\hbar}{2} [(\Omega_R + i \Omega_\alpha)|2\rangle \langle 1| e^{-i\omega t} + (\Omega_R - i \Omega_\alpha)|1\rangle \langle 2| e^{i\omega t} + \omega_0 |2\rangle \langle 2| - |1\rangle \langle 1|]]\). This implies two lasers with the same frequency, orthogonal polarization, and differently shaped time-dependent intensities. Instead, when \(H = H_1; \) \(H = \frac{\hbar}{2} [i \Omega_\alpha|2\rangle \langle 1| e^{-i\omega t} - i \Omega_\alpha|1\rangle \langle 2| e^{i\omega t} + \omega_0 |2\rangle \langle 2| - |1\rangle \langle 1|]]\), which requires only one interaction and level shift engineering so that \(\Delta = 0.\) In this case the \(\pm i\) factors in Eq. (3) can be dropped, which amounts to redefining the states with constant phase factors or to performing an axis rotation, without altering the population transfer.

For the AE scheme the population of the excited state \(P_2\) starting from the ground state \(P_2\) depends on the dimensionless parameters \(\tau = t_0 \beta\) and \(\omega = \Omega_0 / \beta [20] = 1 - \text{sech}^2(2\tau / \pi) \cos^2(\tau(\omega^2 - 4\pi^2 / \pi^2)^{1/2})\). A population transfer near to 1 \((P_2 > 0.999)\) and stable with respect to parameter variations is achieved for \(\omega \approx 3\) and \(\tau \approx 3\). We may calculate \(\Omega_\alpha\) and the minimal time for which the maximum of \(\Omega_\alpha\) with respect to \(t\) is \(\leq \Omega_0\). In the stated range this is accurately given by \(\tau_m = \pi / (4 \omega), \) or \(t_m = \pi / (4 \Omega_0)\). The reduction factor with respect to the adiabatic time \(t_a = 3\) may be very significant, \(t_0 / t_a = \pi / (12 \omega)\); this is 0.09 for \(\omega = 3\) or 0.01 for \(\omega = 20\). Of course the SHAPE \(H_1\) may also drive the system along the adiabatic path outside the \(w, \tau > 3\) domain as illustrated in Fig. 1.

For comparison, the population of the excited state due to a square \(\pi\) pulse with on-resonance Rabi frequency \(\Omega_0\) is \(P_2 = \Omega_0^2 \sin^2(\Omega_0^2 t_m)\). Complete population transfer requires \(\Delta = 0\) and a pulse time \(t_P = \pi / \Omega_0\). For the same \(\Omega_0\) and limiting the auxiliary laser by \(|\Omega_\alpha| \leq \Omega_0\), the minimal characteristic time \(t_0/m\) of the SHAPE method is
by
with SHAPE (AE scheme for ordinary AE adiabatic passage (dotted black line), adiabatic energies (dot-dashed blue lines), average energy evolving with $H_0^\text{t}$ (solid purple line), average energy evolving with $\mathcal{H}_t = H_0^\text{t}$ or $\mathcal{H}_t = H_0^\text{t} + H_1^\text{t}$ (dotted red line), indistinguishable from the adiabatic energy. Parameters: $\omega = 5$ and $\tau = 1.22$.

$t_{0,m} = t_\beta/4$. The actual interaction time to implement a successful population inversion with the AE scheme (SHAPE corrected or not) should be a few times $t_\beta$; this may be estimated from the dependence of the excited population of the adiabatic state with time, which is may be estimated from the dependence of the excited population of the adiabatic state with time, which is $>0.999$ for $t \approx 8t_0$. All calculations are solved numerically by solving the dynamical equation of $\psi^\text{t}$ by a Runge-Kutta method with adaptive step size control.

Figure 2 shows examples of the fidelity ($F = P_2$) with respect to variations in the Rabi frequency and detuning with SHAPE (AE scheme for $H_0^\text{t}$), the evolution with $H_0^\text{t}$ (AE scheme), a Rabi $\pi$ pulse, and a composite $\tilde{\xi}(x)\pi(y) \times \tilde{\xi}(x)$ pulse, a fault-tolerant combination where $x, y$ refer to the laser polarization (and Pauli matrix) involved. Clearly SHAPE provides a fast, robust, and efficient population inversion compared to all other methods. All cases are for the same $\Omega_0$, and in SHAPE $|\Omega_a| \leq \Omega_0$.

**Stimulated Raman adiabatic passage.—**Similar ideas can be applied to three-level STIRAP. The (RWA) Hamiltonian for the two-photon resonance case in a laser-adapted (ordinary) interaction picture reads [7]

$$H_0^\text{t} = \frac{\hbar}{2} \begin{pmatrix} 0 & \Omega_p(t) & 0 \\ \Omega_p(t) & 2\Delta & \Omega_s(t) \\ 0 & \Omega_s(t) & 0 \end{pmatrix},$$

in terms of the Rabi frequencies for the Stokes, $\Omega_s(t)$, and pumping lasers, $\Omega_p(t)$, and the (now constant) laser detuning $\hbar \Delta = (E_2 - E_1) - \hbar \omega_p = (E_2 - E_3) - \hbar \omega_s$, $\omega_p$ and $\omega_s$ being the corresponding laser frequencies. The instantaneous eigenstates $|\lambda_n\rangle$ are $|\lambda_+(t)\rangle = \sin \theta \sin \phi |1\rangle + \cos \phi |2\rangle + \cos \theta \sin \phi |3\rangle$, $|\lambda_-(t)\rangle = \sin \theta \cos \phi |1\rangle - \sin \phi |2\rangle + \cos \theta \cos \phi |3\rangle$, and $|\lambda_0(t)\rangle = \cos \theta |1\rangle - \sin \theta |3\rangle$, with eigenvalues $E_+(t) = \hbar \Omega \cot \phi(t) / 2$, $E_0 = 0$, and $E_-(t) = -\hbar \Omega \tan \phi(t) / 2$. The mixing angles $\theta$ and $\phi$ are, respectively, defined by $\tan \theta = \Omega_p(t) / \Omega_s(t)$ and $\tan(2\phi) = \Omega / (\Delta(t))$, whereas now $\Omega = \sqrt{\Omega_p^2(t) + \Omega_s^2(t)}$.

The population transfer $1 \rightarrow 3$ is realized by the “dark state” $|\lambda_0\rangle$. The Hamiltonian $H_1^\text{t}$, see Eq. (1), takes the form

$$H_1^\text{t} = i\hbar \begin{pmatrix} 0 & \phi \sin \theta & \dot{\theta} \\ -\phi \sin \theta & 0 & -\phi \cos \theta \\ -\dot{\theta} & \phi \cos \theta & 0 \end{pmatrix},$$

with $\dot{\theta} = [\dot{\Omega}_p(t)|\dot{\Omega}_s(t) - \dot{\Omega}_s(t)|\dot{\Omega}_p(t)] / \Omega_0^2$ and $\dot{\phi} = [(\dot{\Omega}_p(t)|\dot{\Omega}_p(t) + \dot{\Omega}_s(t)|\dot{\Omega}_s(t)] / [2\Omega_0^2(\Delta^2 + \Lambda^2)]$. We would need, in principle, three new interactions to implement this Hamiltonian. The laser fields connecting levels 1-2 and 2-3 should have the same frequency as the original ones, and the field connecting levels 1-3 should be on resonance with this transition. (The RWA approximation is assumed in all cases.) If the transition 1-3 is electric-dipole forbidden, as it is the case for hyperfine levels of alkali atoms, a magnetic dipole transition may be used instead, although this may limit the intensity and thus the ability to shorten the times. By working in the adiabatic basis we see that $d|\lambda_0(t)|\psi^\text{t}(t) / dt$ does not depend on $\phi$ for an arbitrary $\psi^\text{t}$, so that the 1-3 and 2-3 auxiliary fields do not affect $|\lambda_0(t)|\psi^\text{t}(t)$ and are thus not necessary for a full passage from 1 to 3. $H_1^\text{t}$ may therefore be simplified to

![FIG. 1](color online). Dimensionless energies (in quasi-interaction picture) $e = E/(\hbar \beta)$ versus dimensionless time $t = t\beta$ for the AE scheme: diabatic energies (dot-dashed blue lines), adiabatic energies (dashed orange lines), average energy evolving with $H_0^\text{t}$ (solid purple line), average energy evolving with $\mathcal{H}_t = H_0^\text{t}$ or $\mathcal{H}_t = H_0^\text{t} + H_1^\text{t}$ (dotted red line), indistinguishable from the adiabatic energy. Parameters: $\omega = 5$ and $\tau = 1.22$.

![FIG. 2](color online). Fidelity ($F = P_2$) versus the changes of (a) the Rabi frequency from $\Omega_0$ to $\Omega_0(1 + \tilde{\eta})$, (b) the detuning by $\delta$. SHAPE, AE scheme (solid red line) with $\Omega_a \ll \Omega_0$, ordinary AE adiabatic passage (dotted black line), Rabi pulse (dashed blue line), composite pulse $\tilde{\xi}(x)\pi(y) \times \tilde{\xi}(x)$ (dot-dashed purple line). $\Omega_0 = 2\pi \times 5$ MHz, $t_0 = 25$ ns, $\beta = 2\pi$ MHz.

![FIG. 3](color online). Level scheme (left) and evolution of Rabi frequencies (right) in a STIRAP sequence of laser pulses, $\tilde{\Omega}(t)$, with $\tilde{\Omega}(t) = 2\pi \times 2$ MHz, $\Delta = 2\pi \times 0.1$ MHz, and $T = 4 \mu s, \tau = 0.1 T$. $\Omega_i / \Omega_0$ (dashed blue line), $\Omega_p / \Omega_0$ (solid red line), $\Omega_a / \Omega_0$ (dotted orange line).
variations. The SHAPE method is also compatible with ing passage 

one can generate be kept real without affecting the results. For alkali atoms 

gineering. Figures 

implementation does not require here any level shift en-

where adiabaticity breaks down; see Fig. 3.

Figures 3 and 4 show examples where [21]

\[
\begin{align*}
\Omega_p(t) &= \Omega_{0f}(t - \tau) ; \\
\Omega_i(t) &= \Omega_{0f}(t) ;
\end{align*}
\]

\[
\begin{cases}
\sin^4(\pi t/T) & (0 < t < T) \\
0 & \text{(otherwise)}
\end{cases}
\]

Figure 3 shows a STIRAP Stokes-pump pulse sequence where adiabaticity breaks down; see Fig. 4(a). Complete transfer can be achieved using the interaction in (6); see Figs. 3 and 4(b). With our parameters the maximum of \(\tilde{\Omega}_a/(2\pi)\) is at 159 kHz, which can be reached with available intensities.

Discussion and outlook.—Similar ideas are applicable to speed up adiabatic approaches in quantum information processing [22–25], interferometric methods in superconducting qubits [26] or quantum dots [27], the creation of entangled states [28], and techniques using avoided crossing passage [29], keeping their stability versus parameter variations. The SHAPE method is also compatible with approaches that optimize \(H_0\), such as the quantum brachisto-chronne [23], since, after optimizing the adiabatic process, it leads to even shorter durations.

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